Math 251 X02
Test One

Time: 50 minutes
Total: 25 marks

Name: _____

1. [5 marks] Let $\mathbf{u} = \begin{bmatrix} 5 \\ 1 \\ -1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} -8 \\ 6 \\ 3 \end{bmatrix}$.

a) Calculate $\mathbf{u} \times \mathbf{v}$.

b) Consider the triangle formed by placing $\mathbf{u}$ and $\mathbf{v}$ tail-to-tail. Find the area of the triangle.

c) Consider the plane that has direction vectors $\mathbf{u}$ and $\mathbf{v}$ and passes through $(3, -1, 2)$. Find the general form of the plane.

2. [6 marks] Let $\mathbf{u} = \begin{bmatrix} 3 \\ -2 \\ 7 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 6 \end{bmatrix}$. Find:

a) a vector of length one parallel to $\mathbf{u} - \mathbf{v}$

b) the angle between $\mathbf{u}$ and $\mathbf{v}$

3. [5 marks] Solve using Gauss-Jordan Elimination:

$$\begin{aligned} 3x + \quad 6y + \quad 6z &= 42 \\ 2x + \quad y + \quad 7z &= 19 \\ 17x + 16y + 52z &= 184 \end{aligned}$$

4. [5 marks] Find the distance between the point $P = (1, -3, 6)$ and the plane $x - 3y + 7z = 4$.

5. [4 marks] How many solutions does the system below have? Your answer will depend on the value of k.

$$\left[\begin{array}{cc|c} 1 & k & 1 \\ k & 64 & 8 \end{array}\right]$$