

Math 251 X01  
Test Three

Time: 50 minutes  
Total: 17 marks

Name: \_\_\_\_\_

1. [3 marks] Solve:

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & -2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 & 5 \\ 0 & 4 & -3 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 40 \\ 64 \\ 104 \end{bmatrix}$$

2. [3 marks] Use Cramer's Rule to find  $y$ :

$$\begin{aligned} -y + z &= -8 \\ 4x + 3y + 7z &= 42 \\ 2x + y - 8z &= 5 \end{aligned}$$

3. [3 marks] Find all eigenvectors of  $A = \begin{bmatrix} 6 & 3 \\ -8 & -4 \end{bmatrix}$  corresponding to  $\lambda = 2$

4. [3 marks] The transformation  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  first performs the transformation  $S$  below, then rotates the vector by  $120^\circ$  counter-clockwise. Find the standard matrix for  $T$ .

$$S \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4x - 6y \\ 8x \end{bmatrix}$$

5. [3 marks]  $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 6 & 9 \\ 4 & 8 & 1 \end{bmatrix}$  has RREF =  $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ .

Find a basis for:

a) the column space of  $A$

b) the row space of  $A$

c) the null space of  $A$

6. [2 marks]

a)  $A$  is a  $3 \times 3$  matrix with  $\text{rank}(A)=3$ . Find the RREF of  $A$ .

b)  $B$  is a  $3 \times 3$  matrix with  $\text{nullity}(B)=3$ . Find the RREF of  $B$ .