1. [3 marks]

Write $\mathbf{w} = \begin{bmatrix} 29 \\ 163 \\ 188 \end{bmatrix}$ as a linear combination of $\mathbf{u} = \begin{bmatrix} 1 \\ 5 \\ 4 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 2 \\ 11 \\ 12 \end{bmatrix}$,
or show that it is impossible to do so.

$$c_1 \vec{u} + c_2 \vec{v} = \vec{w}$$

$$\begin{array}{cc} c_1 & c_2 & \\ \end{array}$$
$$\left[\begin{array}{cc|c} 1 & 2 & 29 \\ 5 & 11 & 163 \\ 4 & 12 & 188 \end{array}\right]$$

$$\begin{array}{l} \\ R_2 - 5R_1 \\ R_3 - 4R_1 \end{array} \left[\begin{array}{cc|c} 1 & 2 & 29 \\ 0 & 1 & 18 \\ 0 & 4 & 72 \end{array}\right]$$

$$\begin{array}{l} R_1 - 2R_2 \\ \\ R_3 - 4R_2 \end{array} \left[\begin{array}{cc|c} 1 & 0 & -7 \\ 0 & 1 & 18 \\ 0 & 0 & 0 \end{array}\right]$$

$$c_1 = -7, \quad c_2 = 18$$

$$-7\vec{u} + 18\vec{v} = \vec{w}$$

2. [3 marks] Write down the system of equations you would use to solve the following problem. **Do not solve the system.**

Balance $C_8H_{18} + O_2 \rightarrow CO_2 + H_2O$

$$w(C_8H_{18}) + x\,O_2 \rightarrow y\,CO_2 + z\,H_2O$$

C : $8w = y$

H : $18w = 2z$

O : $2x = 2y + z$

OR

$$\begin{array}{cccc} w & x & y & z \end{array}$$

$$\left[\begin{array}{cccc|c} 8 & 0 & -1 & 0 & 0 \\ 18 & 0 & 0 & -2 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{array}\right]$$

3. [4 marks] Compute $C^2 - AB^T$ where:

$A = \begin{bmatrix} 1 & -1 \\ 3 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 8 & 7 \\ 2 & -2 \end{bmatrix}$ and $C = \begin{bmatrix} 5 & -3 \\ 9 & 2 \end{bmatrix}$

$$C^2 - AB^T = \begin{bmatrix} 5 & -3 \\ 9 & 2 \end{bmatrix}\begin{bmatrix} 5 & -3 \\ 9 & 2 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 3 & 6 \end{bmatrix}\begin{bmatrix} 8 & 2 \\ 7 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -21 \\ 63 & -23 \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ 66 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -25 \\ -3 & -17 \end{bmatrix}$$

4. [3 marks] Write $A = \begin{bmatrix} 4 & 8 \\ 0 & 1 \end{bmatrix}$ as a product of elementary matrices.

$$\begin{bmatrix} 4 & 8 \\ 0 & 1 \end{bmatrix}$$

$\frac{R_1}{4}$ $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ $E_1 = \begin{bmatrix} 1/4 & 0 \\ 0 & 1 \end{bmatrix}$ $E_1^{-1} = \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}$

$R_1 - 2R_2$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $E_2 = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$ $E_2^{-1} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$$\underbrace{E_2 E_1}_{A^{-1}} A = I$$

$$A^{-1} = E_2 E_1$$

$$A = (E_2 E_1)^{-1}$$

$$= E_1^{-1} E_2^{-1}$$

$$= \begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

5. [3 marks] Find a 2×2 matrix A such that:

a) $A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ but $A \neq \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \checkmark$$

b) $A \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 9 & -2 \end{bmatrix}$$