1. $A = \begin{bmatrix} 1 & 3 \\ -2 & 6 \end{bmatrix}$ Section 4.3

$$\det(A - \lambda I) = \begin{vmatrix} 1-\lambda & 3 \\ -2 & 6-\lambda \end{vmatrix}$$

$$= (1-\lambda)(6-\lambda) + 6$$

$$= \lambda^2 - 7\lambda + 6 + 6$$

$$= \lambda^2 - 7\lambda + 12$$ characteristic polynomial

$$= (\lambda - 3)(\lambda - 4)$$

$$\lambda^2 - 7\lambda + 12 = 0$$

$$(\lambda - 3)(\lambda - 4) = 0$$

$$\lambda = 3, 4$$ eigenvalues of A

E_3: $[A - 3I \mid \vec{0}]$

$$\left[\begin{array}{cc|c} -2 & 3 & 0 \\ -2 & 3 & 0 \end{array}\right]$$

$\frac{R_1}{-2}$ $\left[\begin{array}{cc|c} 1 & -\frac{3}{2} & 0 \\ -2 & 3 & 0 \end{array}\right]$

$R_2 + 2R_1$ $\left[\begin{array}{cc|c} 1 & -\frac{3}{2} & 0 \\ 0 & 0 & 0 \end{array}\right]$

$x_2 = t$

$x_1 - \frac{3}{2} x_2 = 0 \Rightarrow x_1 = \frac{3}{2} t$

$\vec{x} = \begin{bmatrix} 3/2 \\ 1 \end{bmatrix} t$ or $\vec{x} = \begin{bmatrix} 3 \\ 2 \end{bmatrix} t$ Basis for $E_3 = \left\{ \begin{bmatrix} 3 \\ 2 \end{bmatrix} \right\}$

(1) Cont'd

E_4: $[A - 4I \mid \vec{0}]$

$$\left[\begin{array}{cc|c} -3 & 3 & 0 \\ -2 & 2 & 0 \end{array}\right]$$

$$\xrightarrow{\frac{R_1}{-3}} \left[\begin{array}{cc|c} 1 & -1 & 0 \\ -2 & 2 & 0 \end{array}\right]$$

$$\xrightarrow{R_2 + 2R_1} \left[\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$x_2 = t$

$x_1 - x_2 = 0 \Rightarrow x_1 = t$

$\vec{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} t$ Basis for $E_4 = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$

λ	Algebraic Multiplicity = exponent on relevant factor in characteristic polynomial	Geometric Multiplicity = # vectors in basis for E_λ
3	1	1
4	1	1

(3) $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$

$$\det(A - \lambda I) = \begin{vmatrix} 1-\lambda & 1 & 0 \\ 0 & -2-\lambda & 1 \\ 0 & 0 & 3-\lambda \end{vmatrix}$$

$$= (1-\lambda)(-2-\lambda)(3-\lambda)$$ No need to expand it. This is the characteristic polynomial.

$$(1-\lambda)(-2-\lambda)(3-\lambda) = 0$$

$$\lambda = 1, -2, 3$$ eigenvalues of A

E_1: $[A - I \mid \vec{0}]$

$$\left[\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 0 & -3 & 1 & 0 \\ 0 & 0 & 2 & 0 \end{array}\right]$$

$R_2 + 3R_1$ $$\left[\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 \end{array}\right]$$

$R_3 - 2R_2$ $$\left[\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 = t$ $x_2 = 0$ $x_3 = 0$

$\vec{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} t$ Basis for $E_1 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$

③ Cont'd

E_{-2} : $[A + 2I \mid \vec{0}]$

$$\left[\begin{array}{ccc|c} 3 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 5 & 0 \end{array}\right]$$

$$\frac{R_1}{3} \quad \left[\begin{array}{ccc|c} 1 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 5 & 0 \end{array}\right]$$

$$R_3 - 5R_2 \quad \left[\begin{array}{ccc|c} 1 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x_2 = t$

$x_1 + \frac{1}{3}x_2 = 0 \Rightarrow x_1 = \frac{-1}{3}t$

$x_3 = 0$

$$\vec{x} = \begin{bmatrix} \frac{-1}{3} \\ 1 \\ 0 \end{bmatrix} t \quad \text{or} \quad \begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix} t$$

Basis for $E_{-2} = \left\{ \begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix} \right\}$

E_3 : $[A - 3I \mid \vec{0}]$

$$\left[\begin{array}{ccc|c} -2 & 1 & 0 & 0 \\ 0 & -5 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\frac{R_1}{-2} \quad \left[\begin{array}{ccc|c} 1 & \frac{-1}{2} & 0 & 0 \\ 0 & -5 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\rightarrow$

③ Cont'd

$$\xrightarrow{\frac{R_2}{-5}} \left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & 0 & 0 \\ 0 & 1 & -\frac{1}{5} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\xrightarrow{R_1 + \frac{1}{2}R_2} \left[\begin{array}{ccc|c} 1 & 0 & -\frac{1}{10} & 0 \\ 0 & 1 & -\frac{1}{5} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\uparrow$$

$$x_3 = t$$

$$x_1 - \frac{1}{10}x_3 = 0 \Rightarrow x_1 = \frac{1}{10}t$$

$$x_2 - \frac{1}{5}x_3 = 0 \Rightarrow x_2 = \frac{1}{5}t$$

$$\vec{x} = \begin{bmatrix} \frac{1}{10} \\ \frac{1}{5} \\ 1 \end{bmatrix} t \quad \text{or} \quad \begin{bmatrix} 1 \\ 2 \\ 10 \end{bmatrix} t$$

Basis for $E_3 = \left\{ \begin{bmatrix} 1 \\ 2 \\ 10 \end{bmatrix} \right\}$

λ	Alg. Mult.	Geo. Mult.
1	1	1
−2	1	1
3	1	1

(5) $A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & -1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

$$\det(A - \lambda I) = \begin{vmatrix} 1-\lambda & 2 & 0 \\ -1 & -1-\lambda & 1 \\ 0 & 1 & 1-\lambda \end{vmatrix}$$

$$= (1-\lambda)\begin{vmatrix} -1-\lambda & 1 \\ 1 & 1-\lambda \end{vmatrix} - 2\begin{vmatrix} -1 & 1 \\ 0 & 1-\lambda \end{vmatrix}$$

$$= (1-\lambda)\left[(-1-\lambda)(1-\lambda) - 1\right] - 2\left[-1+\lambda\right]$$

$$= (1-\lambda)\left[\lambda^2 - 2\right] + 2(1-\lambda)$$

$$= (1-\lambda)\left[\lambda^2 - 2 + 2\right]$$

$$= (1-\lambda)\lambda^2$$

No need to expand it.

This is the characteristic polynomial.

$$(1-\lambda)\lambda^2 = 0$$

$\lambda = 0, 0, 1$ eigenvalues of A

E_0: $[A - 0I \mid \vec{0}]$

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ -1 & -1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

(5) Cont'd

$$R_2 + R_1 \quad \left[\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

$$\begin{array}{l} R_1 - 2R_2 \\ \\ R_3 - R_2 \end{array} \quad \left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$x_3 = t$$

$$x_1 - 2x_3 = 0 \Rightarrow x_1 = 2t$$

$$x_2 + x_3 = 0 \Rightarrow x_2 = -t$$

$$\bar{x} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} t \quad \text{Basis for } E_0 = \left\{ \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \right\}$$

E_1: $[A - I \mid \vec{0}]$

$$\left[\begin{array}{ccc|c} 0 & 2 & 0 & 0 \\ -1 & -2 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right]$$

$$\text{Reorder rows} \quad \left[\begin{array}{ccc|c} -1 & -2 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \end{array}\right]$$

$$\frac{R_1}{-1} \quad \left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \end{array}\right]$$

$$\begin{array}{l} R_1 - 2R_2 \\ \\ R_3 - 2R_2 \end{array} \quad \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$x_3 = t$$

$$x_1 - x_3 = 0 \Rightarrow x_1 = t$$

$$x_2 = 0$$

$\rightarrow$

(5) Cont'd

$$\vec{x} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} t \quad \text{Basis for } E_1 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$$

λ	Alg. Mult.	Geo. Mult.
0	2	1
1	1	1

(9) $A = \begin{bmatrix} 3 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

$$\det(A - \lambda I) = \begin{vmatrix} 3-\lambda & 1 & 0 & 0 \\ -1 & 1-\lambda & 0 & 0 \\ 0 & 0 & 1-\lambda & 4 \\ 0 & 0 & 1 & 1-\lambda \end{vmatrix}$$

$$= (3-\lambda) \begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & 1-\lambda & 4 \\ 0 & 1 & 1-\lambda \end{vmatrix} - 1 \begin{vmatrix} -1 & 0 & 0 \\ 0 & 1-\lambda & 4 \\ 0 & 1 & 1-\lambda \end{vmatrix}$$

$$= (3-\lambda)(1-\lambda) \begin{vmatrix} 1-\lambda & 4 \\ 1 & 1-\lambda \end{vmatrix} + 1 \begin{vmatrix} 1-\lambda & 4 \\ 1 & 1-\lambda \end{vmatrix}$$

$$= (3-\lambda)(1-\lambda)\left[\underbrace{(1-\lambda)^2 - 4}\right] + 1\left[\underbrace{(1-\lambda)^2 - 4}\right]$$

Factor

$$= \left[(3-\lambda)(1-\lambda) + 1\right]\left[\underbrace{(1-\lambda)^2 - 4}\right]$$

$$= \left[\lambda^2 - 4\lambda + 4\right]\left[\lambda^2 - 2\lambda - 3\right]$$

$$= (\lambda - 2)^2 (\lambda - 3)(\lambda + 1)$$ No need to expand.

This is the characteristic polynomial.

$$(\lambda - 2)^2 (\lambda - 3)(\lambda + 1) = 0$$

$\lambda = -1, 2, 3$ eigenvalues of A.

(9) Cont'd

E_{-1}: $[A + I \mid \vec{0}]$

$$\left[\begin{array}{cccc|c} 4 & 1 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 4 & 0 \\ 0 & 0 & 1 & 2 & 0 \end{array}\right]$$

$R_1 \leftrightarrow R_2$ Then $R_1/-1$

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 0 & 0 \\ 4 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 4 & 0 \\ 0 & 0 & 1 & 2 & 0 \end{array}\right]$$

$R_2 - 4R_1$

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 0 & 0 \\ 0 & 9 & 0 & 0 & 0 \\ 0 & 0 & 2 & 4 & 0 \\ 0 & 0 & 1 & 2 & 0 \end{array}\right]$$

$\frac{R_2}{9}$ and $R_3 \leftrightarrow R_4$

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 2 & 4 & 0 \end{array}\right]$$

$R_1 + 2R_2$

$R_4 - 2R_3$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$x_4 = t$

$x_1 = 0$

$x_2 = 0$

$x_3 + 2x_4 = 0 \Rightarrow x_3 = -2t$

$\bar{x} = \begin{bmatrix} 0 \\ 0 \\ -2 \\ 1 \end{bmatrix} t$ Basis for $E_{-1} = \left\{ \begin{bmatrix} 0 \\ 0 \\ -2 \\ 1 \end{bmatrix} \right\}$

(9) Cont'd

E_2: $[A - 2I \mid \vec{0}]$

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 4 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array}\right]$$

$R_2 + R_1$ and $R_3 \leftrightarrow R_4$

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 4 & 0 \end{array}\right]$$

$R_4 + R_3$

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 3 & 0 \end{array}\right]$$

$\frac{R_4}{3}$

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$R_3 + R_4$ then reorder rows

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$\uparrow$

$x_2 = t$

$x_1 + x_2 = 0 \Rightarrow x_1 = -t$

$x_3 = 0$

$x_4 = 0$

$$\vec{x} = \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} t \quad \text{or} \quad \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} t$$

Basis for $E_2 = \left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} \right\}$

(9) Cont'd

E_3: $[A - 3I \mid \bar{0}]$

$$\left[\begin{array}{cccc|c} 0 & 1 & 0 & 0 & 0 \\ -1 & -2 & 0 & 0 & 0 \\ 0 & 0 & -2 & 4 & 0 \\ 0 & 0 & 1 & -2 & 0 \end{array}\right]$$

$R_2 \leftrightarrow R_1$ and $R_3 \leftrightarrow R_4$

$$\left[\begin{array}{cccc|c} -1 & -2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & -2 & 4 & 0 \end{array}\right]$$

$\frac{R_1}{-1}$

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & -2 & 4 & 0 \end{array}\right]$$

$R_1 - 2R_2$

$R_4 + 2R_3$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$x_4 = t$

$x_1 = 0$
$x_2 = 0$
$x_3 - 2x_4 = 0 \Rightarrow x_3 = 2t$

$\bar{x} = \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix} t$ Basis for $E_3 = \left\{ \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix} \right\}$

λ	Alg. Mult.	Geo. Mult.
-1	1	1
2	2	1
3	1	1

(15) Let $\vec{x} = c_1\vec{v}_1 + c_2\vec{v}_2$

$$c_1\begin{bmatrix}1\\-1\end{bmatrix} + c_2\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}5\\1\end{bmatrix}$$

$$\begin{array}{cc} c_1 & c_2 \\ \end{array}$$
$$\left[\begin{array}{cc|c} 1 & 1 & 5 \\ -1 & 1 & 1 \end{array}\right]$$

$$R_2+R_1 \quad \left[\begin{array}{cc|c} 1 & 1 & 5 \\ 0 & 2 & 6 \end{array}\right]$$

$$\frac{R_2}{2} \quad \left[\begin{array}{cc|c} 1 & 1 & 5 \\ 0 & 1 & 3 \end{array}\right]$$

$$R_1-R_2 \quad \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 3 \end{array}\right]$$

$$c_1 = 2, \quad c_2 = 3$$

So $\vec{x} = 2\vec{v}_1 + 3\vec{v}_2$

$$\begin{aligned} A^{10}\vec{x} &= A^{10}(2\vec{v}_1 + 3\vec{v}_2) \\ &= A^{10}(2\vec{v}_1) + A^{10}(3\vec{v}_2) \\ &= 2A^{10}\vec{v}_1 + 3A^{10}\vec{v}_2 \\ &= 2\left(\tfrac{1}{2}\right)^{10}\vec{v}_1 + 3(2)^{10}\vec{v}_2 \qquad \boxed{\text{Using } A^n\vec{v} = \lambda^n\vec{v}} \\ &= 2^{-9}\begin{bmatrix}1\\-1\end{bmatrix} + 3\cdot 2^{10}\begin{bmatrix}1\\1\end{bmatrix} \\ &= \begin{bmatrix} 2^{-9} + 3\cdot 2^{10} \\ -2^{-9} + 3\cdot 2^{10} \end{bmatrix} \end{aligned}$$

(22) Show that $(A - cI)\bar{v} = (\lambda - c)\bar{v}$,
given that $A\bar{v} = \lambda\bar{v}$.

$$\begin{aligned}(A - cI)\bar{v} &= A\bar{v} - cI\bar{v} \\ &= \lambda\bar{v} - c\bar{v} \\ &= (\lambda - c)\bar{v}\end{aligned}$$

(23) a) $\det(A - \lambda I) = \begin{vmatrix} 3-\lambda & 2 \\ 5 & -\lambda \end{vmatrix}$

$$\begin{aligned}&= (3-\lambda)(-\lambda) - 10 \\ &= -3\lambda + \lambda^2 - 10 \\ &= \lambda^2 - 3\lambda - 10 \\ &= (\lambda - 5)(\lambda + 2)\end{aligned}$$

Set $\det(A - \lambda I) = 0$: $(\lambda - 5)(\lambda + 2) = 0$
$\lambda = 5, -2$

E_5: $[A - 5I \mid \vec{0}]$

$$\left[\begin{array}{cc|c} -2 & 2 & 0 \\ 5 & -5 & 0 \end{array}\right]$$

$$\frac{R_1}{-2} \left[\begin{array}{cc|c} 1 & -1 & 0 \\ 5 & -5 & 0 \end{array}\right]$$

$$R_2 - 5R_1 \left[\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$x_2 = t$

$x_1 - x_2 = 0 \Rightarrow x_1 = t$ $\rightarrow$

(23) Cont'd $\bar{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} t$ Basis for $E_5 = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$

E_{-2}: $[A + 2I \mid \bar{0}]$

$$\left[\begin{array}{cc|c} 5 & 2 & 0 \\ 5 & 2 & 0 \end{array}\right]$$

$$\frac{R_1}{5} \quad \left[\begin{array}{cc|c} 1 & \frac{2}{5} & 0 \\ 5 & 2 & 0 \end{array}\right]$$

$$R_2 - 5R_1 \quad \left[\begin{array}{cc|c} 1 & \frac{2}{5} & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$x_2 = t$

$x_1 + \frac{2}{5} x_2 = 0 \Rightarrow x_1 = -\frac{2}{5} t$

$\bar{x} = \begin{bmatrix} -\frac{2}{5} \\ 1 \end{bmatrix} t$ or $\begin{bmatrix} 2 \\ -5 \end{bmatrix} t$

Basis for $E_{-2} = \left\{ \begin{bmatrix} 2 \\ -5 \end{bmatrix} \right\}$

b)

Matrix	Eigenvalues	Basis for Eigenspaces
A	$5, -2$	$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ -5 \end{bmatrix}$ respectively
A^{-1}	$\frac{1}{5}, \frac{-1}{2}$	same as above
$A - 2I$	$3, -4$	same as above
$A + 2I$	$7, 0$	same as above