

$$\begin{aligned} \textcircled{1} \quad T_A(\vec{u}) &= A\vec{u} \\ &= \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} \\ &= \begin{bmatrix} 0 \\ 11 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} T_A(\vec{v}) &= A\vec{v} \\ &= \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \end{bmatrix} \\ &= \begin{bmatrix} 8 \\ 1 \end{bmatrix} \end{aligned}$$

⑨ Show that T is not a matrix transformation.

No matrix M exists so that

$$M \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} xy \\ x+y \end{bmatrix}$$

Note: M cannot have variables in it.

$$\textcircled{11} \quad T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x+y \\ x-y \end{bmatrix}$$

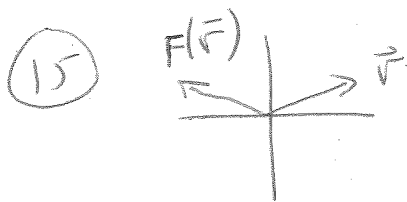
$$\Rightarrow T \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \text{and} \quad T \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\Rightarrow [T] = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\textcircled{13} \quad T \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x-y+z \\ 2x+y-3z \end{bmatrix}$$

$$\Rightarrow T \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad T \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \quad T \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

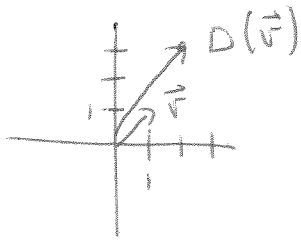
$$\Rightarrow [T] = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \end{bmatrix}$$



$$F \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \quad \text{and} \quad F \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$[F] = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

(17)



$$D \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \quad D \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

$$[D] = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

(21)

$$\theta = 30^\circ \text{ clockwise} \\ = -30^\circ$$

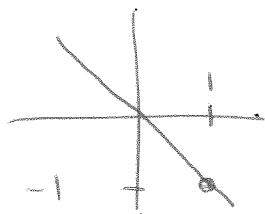
$$[T] = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \theta = -30^\circ$$

$$= \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

$$\text{or } \frac{1}{2} \begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{3} \end{bmatrix}$$

$$\begin{aligned} \cos 30^\circ &= \frac{\sqrt{3}}{2} \\ \cos(-30^\circ) &= \frac{\sqrt{3}}{2} \\ \sin 30^\circ &= \frac{1}{2} \\ \sin(-30^\circ) &= -\frac{1}{2} \end{aligned}$$

(23) Projection onto the line $y = -x$.

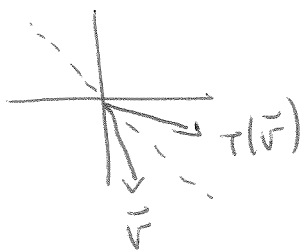


the line has $\vec{d} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$$[T] = \frac{1}{a^2 + b^2} \begin{bmatrix} a^2 & ab \\ ab & b^2 \end{bmatrix} \quad \text{where} \quad \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

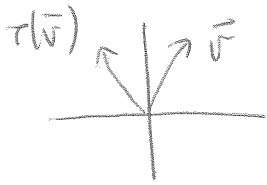
(25) Reflection in the line $y = -x$



$$T \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \quad T \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$[T] = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

(37)

 T_1 : reflection in y-axis

$$T_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \quad T_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$[T_1] = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

T_2 : clockwise rotation by 30°
 = rotation by -30°

$$[T_2] = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}_{\theta = -30^\circ}$$

$$= \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

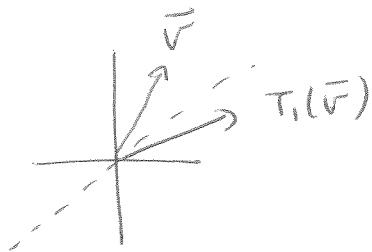
$$= \frac{1}{2} \begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{3} \end{bmatrix}$$

$$[T_2 \circ T_1] = [T_2][T_1]$$

$$= \frac{1}{2} \begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{3} \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} -\sqrt{3} & 1 \\ 1 & \sqrt{3} \end{bmatrix}$$

(39)

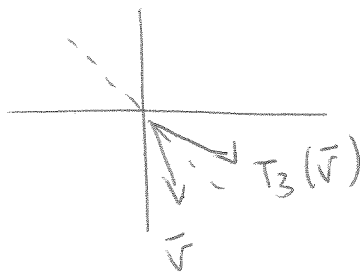
 T_1 : reflection in $y=x$ T_2 : rotation by 30° T_3 : reflection in $y=-x$ 

$$T_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad T_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$[T_1] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$[T_2] = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \quad \theta = 30^\circ$$

$$= \frac{1}{2} \begin{bmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{bmatrix}$$



$$T_3 \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \quad T_3 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$[T_3] = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

Standard matrix is $[T_3][T_2][T_1]$

$$= \frac{1}{2} \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} -1 & -\sqrt{3} \\ -\sqrt{3} & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} -\sqrt{3} & -1 \\ 1 & -\sqrt{3} \end{bmatrix}$$