

①

$$LU\bar{x} = \bar{b}$$

$$\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ 0 & 6 \end{bmatrix} \bar{x} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

First solve  $L\bar{y} = \bar{b}$

$$\begin{array}{cc|c} y_1 & y_2 & \\ \hline 1 & 0 & 5 \\ -1 & 1 & 1 \end{array}$$

$$y_1 = 5$$

$$-y_1 + y_2 = 1 \Rightarrow -5 + y_2 = 1 \Rightarrow y_2 = 6$$

Then solve  $U\bar{x} = \bar{y}$

$$\begin{array}{cc|c} x_1 & x_2 & \\ \hline -2 & 1 & 5 \\ 0 & 6 & 6 \end{array}$$

$$6x_2 = 6 \Rightarrow x_2 = 1$$

$$-2x_1 + x_2 = 5 \Rightarrow -2x_1 + 1 = 5 \Rightarrow x_1 = -2$$

$$\bar{x} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

③

$$LU\bar{x} = \bar{b}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & -\frac{5}{4} & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & -2 \\ 0 & 4 & -6 \\ 0 & 0 & -\frac{7}{2} \end{bmatrix} \bar{x} = \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}$$

First solve  $L\bar{y} = \bar{b}$

$$\begin{array}{ccc|c} y_1 & y_2 & y_3 & \\ \hline 1 & 0 & 0 & -3 \\ -1 & 1 & 0 & 1 \\ 2 & -\frac{5}{4} & 1 & 0 \end{array}$$

$$y_1 = -3$$

$$-y_1 + y_2 = 1 \Rightarrow 3 + y_2 = 1 \Rightarrow y_2 = -2$$

$$2y_1 - \frac{5}{4}y_2 + y_3 = 0 \Rightarrow -6 + \frac{5}{2} + y_3 = 0 \Rightarrow y_3 = \frac{7}{2}$$

Then solve  $U\bar{x} = \bar{y}$

$$\begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \hline 2 & 1 & -2 & -3 \\ 0 & 4 & -6 & -2 \\ 0 & 0 & -\frac{7}{2} & \frac{7}{2} \end{array}$$

$$-\frac{7}{2}x_3 = \frac{7}{2} \Rightarrow x_3 = -1$$

$$4x_2 - 6x_3 = -2 \Rightarrow 4x_2 + 6 = -2 \Rightarrow x_2 = -2$$

$$2x_1 + x_2 - 2x_3 = -3 \Rightarrow 2x_1 - 2 + 2 = -3 \Rightarrow x_1 = -\frac{3}{2}$$

$$\bar{x} = \begin{bmatrix} -\frac{3}{2} \\ -2 \\ -1 \end{bmatrix}$$

⑦

$$\begin{bmatrix} 1 & 2 \\ -3 & -1 \end{bmatrix}$$

$R_2 + 3R_1$

$$\begin{bmatrix} 1 & 2 \\ 0 & 5 \end{bmatrix}$$

REF ✓

$$k = -3$$

$$L = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 2 \\ 0 & 5 \end{bmatrix}$$

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$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 8 & 7 & 9 \end{bmatrix}$$

$$\begin{array}{l} R_2 - 4R_1 \\ R_3 - 8R_1 \end{array} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & -9 & -15 \end{bmatrix} \quad \begin{array}{l} k=4 \\ k=8 \end{array}$$

$$R_3 - 3R_2 \quad \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & 3 \end{bmatrix} \quad k=3$$

REF ✓

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 8 & 3 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & 3 \end{bmatrix}$$