Section 3.3

(1) $A = \begin{bmatrix} 4 & 7 \\ 1 & 2 \end{bmatrix}$

$\det A = 1$

$A^{-1} = \frac{1}{1}\begin{bmatrix} 2 & -7 \\ -1 & 4 \end{bmatrix}$

$= \begin{bmatrix} 2 & -7 \\ -1 & 4 \end{bmatrix}$

(5) $A = \begin{bmatrix} 3/4 & 3/5 \\ 5/6 & 2/3 \end{bmatrix}$

$\det A = \frac{3}{4}\left(\frac{2}{3}\right) - \frac{3}{5}\left(\frac{5}{6}\right)$

$= \frac{1}{2} - \frac{1}{2}$

$= 0$

$A^{-1}$ does not exist

(7) $A = \begin{bmatrix} -1.5 & -4.2 \\ 0.5 & 2.4 \end{bmatrix}$

$\det A = -1.5(2.4) + 4.2(0.5)$

$= -1.5$

$= -3/2$

$A^{-1} = -\frac{2}{3}\begin{bmatrix} 2.4 & 4.2 \\ -0.5 & -1.5 \end{bmatrix}$

or $\begin{bmatrix} -1.6 & -2.8 \\ \frac{1}{3} & 1 \end{bmatrix}$

(9) $A = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

$\det A = a^2 + b^2$

$A^{-1} = \dfrac{1}{a^2+b^2}\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$

(11) $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$

$X = A^{-1}B$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix}$

$= \begin{bmatrix} -5 \\ 9 \end{bmatrix}$

(19) Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$(A+B)^{-1} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}^{-1} =$ undefined

$A^{-1} + B^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(23) $ABXA^{-1}B^{-1} = I + A$

Left multiply by $A^{-1}$: $A^{-1}ABXA^{-1}B^{-1} = A^{-1}(I+A)$

$BXA^{-1}B^{-1} = A^{-1}(I+A)$

Left multiply by $B^{-1}$: $B^{-1}BXA^{-1}B^{-1} = B^{-1}A^{-1}(I+A)$

$XA^{-1}B^{-1} = B^{-1}A^{-1}(I+A)$

Right multiply by $B$: $XA^{-1}B^{-1}B = B^{-1}A^{-1}(I+A)B$

$XA^{-1} = B^{-1}A^{-1}(I+A)B$

Right multiply by $A$: $XA^{-1}A = B^{-1}A^{-1}(I+A)BA$

$X = B^{-1}A^{-1}(I+A)BA$

or $X = (B^{-1}A^{-1} + B^{-1}A^{-1}A)BA$

$= (B^{-1}A^{-1} + B^{-1})BA$

$= B^{-1}A^{-1}BA + B^{-1}BA$

$= (AB)^{-1}BA + A$

(31) $E_1 = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$ represents $3R_1$

$E_1^{-1}$ represents $\frac{R_1}{3}$

$E_1^{-1} = \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & 1 \end{bmatrix}$

(33) $E_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ represents $R_1 \leftrightarrow R_2$

$E_1^{-1}$ represents $R_1 \leftrightarrow R_2$

$E_1^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(35) $E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$ represents $R_2 - 2R_3$

$E_1^{-1}$ represents $R_2 + 2R_3$

$E_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

(39) $A = \begin{bmatrix} 1 & 0 \\ -1 & -2 \end{bmatrix}$

$R_2 + R_1$ $\begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$ $\quad E_1 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ $\quad E_1^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$

($R_2 - R_1$ undoes it)

$\frac{R_2}{-2}$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\quad E_2 = \begin{bmatrix} 1 & 0 \\ 0 & -\frac{1}{2} \end{bmatrix}$ $\quad E_2^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$

($-2R_2$ undoes it)

$$\underbrace{E_2 E_1}_{A^{-1}} A = I$$

$$A^{-1} = E_2 E_1$$

$$\begin{aligned} A &= (A^{-1})^{-1} \\ &= (E_2 E_1)^{-1} \\ &= E_1^{-1} E_2^{-1} \end{aligned}$$

Therefore $A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

and $A = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$

(43) a) $A$ invertible and $BA = CA$

$$\Rightarrow \quad BAA^{-1} = CAA^{-1}$$
$$B = C$$

b) Find $A, B, C$ so that $BA = CA$ and $B \neq C$

Let $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$

Then $BA = CA$ and $B \neq C$.

(53) $[A \mid I]$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 3 & 1 & 2 & 0 & 1 & 0 \\ 2 & 3 & -1 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{array}{l} R_2 - 3R_1 \\ R_3 - 2R_1 \end{array} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 4 & -4 & -3 & 1 & 0 \\ 0 & 5 & -5 & -2 & 0 & 1 \end{array}\right]$$

$$\frac{R_2}{4} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & -\frac{3}{4} & \frac{1}{4} & 0 \\ 0 & 5 & -5 & -2 & 0 & 1 \end{array}\right]$$

$$R_3 - 5R_2 \left[\begin{array}{ccc|ccc} & & & & & \\ 0 & 0 & 0 & & & \end{array}\right]$$

$A^{-1}$ does not exist

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$[A \mid I]$

$$\left[\begin{array}{ccc|ccc} a & 0 & 0 & 1 & 0 & 0 \\ 1 & a & 0 & 0 & 1 & 0 \\ 0 & 1 & a & 0 & 0 & 1 \end{array}\right]$$

$a = 0$: $A^{-1}$ does not exist

$a \neq 0$:

$$\frac{R_1}{a} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{a} & 0 & 0 \\ 1 & a & 0 & 0 & 1 & 0 \\ 0 & 1 & a & 0 & 0 & 1 \end{array}\right]$$

$$R_2 - R_1 \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{a} & 0 & 0 \\ 0 & a & 0 & -\frac{1}{a} & 1 & 0 \\ 0 & 1 & a & 0 & 0 & 1 \end{array}\right]$$

$$\frac{R_2}{a} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{a} & 0 & 0 \\ 0 & 1 & 0 & -\frac{1}{a^2} & \frac{1}{a} & 0 \\ 0 & 1 & a & 0 & 0 & 1 \end{array}\right]$$

$$R_3 - R_2 \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{a} & 0 & 0 \\ 0 & 1 & 0 & -\frac{1}{a^2} & \frac{1}{a} & 0 \\ 0 & 0 & a & \frac{1}{a^2} & -\frac{1}{a} & 1 \end{array}\right]$$

$$\frac{R_3}{a} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{a} & 0 & 0 \\ 0 & 1 & 0 & -\frac{1}{a^2} & \frac{1}{a} & 0 \\ 0 & 0 & 1 & \frac{1}{a^3} & -\frac{1}{a^2} & \frac{1}{a} \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} \frac{1}{a} & 0 & 0 \\ -\frac{1}{a^2} & \frac{1}{a} & 0 \\ \frac{1}{a^3} & -\frac{1}{a^2} & \frac{1}{a} \end{bmatrix} \quad (a \neq 0)$$