① $A+2D = \begin{bmatrix} 3 & 0 \\ -1 & 5 \end{bmatrix} + 2\begin{bmatrix} 0 & -3 \\ -2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 3 & 0 \\ -1 & 5 \end{bmatrix} + \begin{bmatrix} 0 & -6 \\ -4 & 2 \end{bmatrix}$

$= \begin{bmatrix} 3 & -6 \\ -5 & 7 \end{bmatrix}$

③ $B-C$ is undefined because $B$ and $C$ have different sizes

⑤ $AB = \begin{bmatrix} 3 & 0 \\ -1 & 5 \end{bmatrix}\begin{bmatrix} 4 & -2 & 1 \\ 0 & 2 & 3 \end{bmatrix}$

$= \begin{bmatrix} 12 & -6 & 3 \\ -4 & 12 & 14 \end{bmatrix}$

⑦ $D+BC = \begin{bmatrix} 0 & -3 \\ -2 & 1 \end{bmatrix} + \begin{bmatrix} 4 & -2 & 1 \\ 0 & 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$

$= \begin{bmatrix} 0 & -3 \\ -2 & 1 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ 21 & 26 \end{bmatrix}$

$= \begin{bmatrix} 3 & 3 \\ 19 & 27 \end{bmatrix}$

⑨ $E(AF) = \begin{bmatrix} 4 & 2 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ -1 & 5 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \end{bmatrix}$

$= \begin{bmatrix} 4 & 2 \end{bmatrix}\begin{bmatrix} -3 \\ 11 \end{bmatrix}$

$= \begin{bmatrix} 10 \end{bmatrix}$

(13) $B^T C^T - (CB)^T$

$$= \begin{bmatrix} 4 & 0 \\ -2 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix} - \left( \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} 4 & -2 & 1 \\ 0 & 2 & 3 \end{bmatrix} \right)^T$$

$$= \begin{bmatrix} 4 & 12 & 20 \\ 2 & 2 & 2 \\ 7 & 15 & 23 \end{bmatrix} - \begin{bmatrix} 4 & 2 & 7 \\ 12 & 2 & 15 \\ 20 & 2 & 23 \end{bmatrix}^T$$

$$= \begin{bmatrix} 4 & 12 & 20 \\ 2 & 2 & 2 \\ 7 & 15 & 23 \end{bmatrix} - \begin{bmatrix} 4 & 12 & 20 \\ 2 & 2 & 2 \\ 7 & 15 & 23 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(15) $$A^3 = \underbrace{\begin{bmatrix} 3 & 0 \\ -1 & 5 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ -1 & 5 \end{bmatrix}} \begin{bmatrix} 3 & 0 \\ -1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 0 \\ -8 & 25 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ -1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 27 & 0 \\ -49 & 125 \end{bmatrix}$$

(17) Let $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \checkmark$$

Section 3.1

(19) Let
$$B = \begin{array}{c c} & \begin{array}{ccc} \text{Doohickey} & \text{Gizmo} & \text{Widget} \end{array} \\ \begin{array}{c} \text{Truck} \\ \text{Train} \end{array} & \begin{bmatrix} 1.5 & 1 & 2 \\ 1.75 & 1.5 & 1 \end{bmatrix} \end{array}$$

A was given:
$$A = \begin{array}{c c} & \begin{array}{cc} \text{Warehouse 1} & \text{Warehouse 2} \end{array} \\ \begin{array}{c} D \\ G \\ W \end{array} & \begin{bmatrix} 200 & 75 \\ 150 & 100 \\ 100 & 125 \end{bmatrix} \end{array}$$

$$BA = \begin{array}{c c} & \begin{array}{cc} \text{Warehouse 1} & \text{Warehouse 2} \end{array} \\ \begin{array}{c} \text{Truck} \\ \text{Train} \end{array} & \begin{bmatrix} 650 & 462.5 \\ 675 & 406.25 \end{bmatrix} \end{array}$$

(21)
$$\begin{bmatrix} 1 & -2 & 3 \\ 2 & 1 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

(35) a)
$$A^2 = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix}$$

$$A^3 = A^2 A = \begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$A^4 = A^3 A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix}$$

$\rightarrow$

$$A^5 = A^4 A$$
$$= \begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$$

$$A^6 = A^5 A$$
$$= \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^7 = A^6 A$$
$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}$$

b) We know from part a) that $A^6 = I$

$$\frac{2001}{6} = 333 + \frac{3}{6}$$
$$2001 = 333(6) + 3$$
$$A^{2001} = A^{6(333)+3}$$
$$= A^{6(333)} \cdot A^3$$
$$= (A^6)^{333} \cdot A^3$$
$$= I^{333} \cdot A^3$$
$$= A^3$$

(38) a)

$$A^2 = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta - \sin^2\theta & -\cos\theta\sin\theta - \cos\theta\sin\theta \\ \sin\theta\cos\theta + \sin\theta\cos\theta & -\sin^2\theta + \cos^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta - \sin^2\theta & -2\sin\theta\cos\theta \\ 2\sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix}$$

Identities:

$2\sin\theta\cos\theta = \sin 2\theta$

$\cos^2\theta - \sin^2\theta = \cos 2\theta$

$$= \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$$