(5)

Let x = # bags of house blend
y = " special "
z = " gourmet "

kg of Columbian beans: $0.3x + 0.2y + 0.1z = 30$
" Kenyan " $0.2y + 0.2z = 15$
" French " $0.2x + 0.1y + 0.2z = 25$

$$\begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \left[\begin{array}{ccc|c} 0.3 & 0.2 & 0.1 & 30 \\ 0 & 0.2 & 0.2 & 15 \\ 0.2 & 0.1 & 0.2 & 25 \end{array}\right] \end{array}$$

$$R_1 \leftrightarrow R_3 \quad \left[\begin{array}{ccc|c} 0.2 & 0.1 & 0.2 & 25 \\ 0 & 0.2 & 0.2 & 15 \\ 0.3 & 0.2 & 0.1 & 30 \end{array}\right]$$

$$5R_1 \quad \left[\begin{array}{ccc|c} 1 & 0.5 & 1 & 125 \\ 0 & 0.2 & 0.2 & 15 \\ 0.3 & 0.2 & 0.1 & 30 \end{array}\right]$$

$$R_3 - 0.3R_1 \quad \left[\begin{array}{ccc|c} 1 & 0.5 & 1 & 125 \\ 0 & 0.2 & 0.2 & 15 \\ 0 & 0.05 & -0.2 & -7.5 \end{array}\right]$$

$$5R_2 \quad \left[\begin{array}{ccc|c} 1 & 0.5 & 1 & 125 \\ 0 & 1 & 1 & 75 \\ 0 & 0.05 & -0.2 & -7.5 \end{array}\right]$$

$\rightarrow$

Recopying matrix:

$$\left[\begin{array}{ccc|c} 1 & 0.5 & 1 & 125 \\ 0 & 1 & 1 & 75 \\ 0 & 0.05 & -0.2 & -7.5 \end{array}\right]$$

$$\begin{array}{l} R_1 - 0.5R_2 \\ \\ R_3 - 0.05R_2 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0.5 & 87.5 \\ 0 & 1 & 1 & 75 \\ 0 & 0 & -0.25 & -11.25 \end{array}\right]$$

$$\begin{array}{l} \\ \\ -4R_3 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0.5 & 87.5 \\ 0 & 1 & 1 & 75 \\ 0 & 0 & 1 & 45 \end{array}\right]$$

$$\begin{array}{l} R_1 - 0.5R_3 \\ R_2 - R_1 \end{array} \begin{array}{c} \begin{array}{cccc} x & y & z & \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 0 & 65 \\ 0 & 1 & 0 & 30 \\ 0 & 0 & 1 & 45 \end{array}\right] \end{array}$$

The merchant can make:

65 bags of house blend
30 " special "
45 " gourmet "

⑦ $w\, FeS_2 + x\, O_2 \to y\, Fe_2O_3 + z\, SO_2$

$$\begin{aligned}
Fe:&\quad w = 2y &&\to\quad w - 2y = 0\\
S:&\quad 2w = z &&\to\quad 2w - z = 0\\
O:&\quad 2x = 3y + 2z &&\to\quad 2x - 3y - 2z = 0
\end{aligned}$$

$$\begin{array}{cccc|c} w & x & y & z & \\ \end{array}$$
$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0\\ 2 & 0 & 0 & -1 & 0\\ 0 & 2 & -3 & -2 & 0 \end{array}\right]$$

$R_2 - 2R_1$
$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0\\ 0 & 0 & 4 & -1 & 0\\ 0 & 2 & -3 & -2 & 0 \end{array}\right]$$

$\frac{R_3}{2}$ then $R_2 \leftrightarrow R_3$
$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0\\ 0 & 1 & -3/2 & -1 & 0\\ 0 & 0 & 4 & -1 & 0 \end{array}\right]$$

$\frac{R_3}{4}$
$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0\\ 0 & 1 & -3/2 & -1 & 0\\ 0 & 0 & 1 & -1/4 & 0 \end{array}\right]$$

$R_1 + 2R_3$
$R_2 + \frac{3}{2}R_3$
$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -1/2 & 0\\ 0 & 1 & 0 & -11/8 & 0\\ 0 & 0 & 1 & -1/4 & 0 \end{array}\right]$$

$$z = t$$
$$\begin{cases} w = \frac{t}{2}\\ x = \frac{11t}{8}\\ y = \frac{t}{4} \end{cases}$$

Want smallest non-negative integer solution.

$t = 8 \Rightarrow [w, x, y, z] = [4, 11, 2, 8]$

$4\, FeS_2 + 11\, O_2 \to 2\, Fe_2O_3 + 8\, SO_2$

15 a)

Inflow = Outflow

A: $20 = f_1 + f_2 \rightarrow f_1 + f_2 = 20$

B: $10 + f_2 = f_3 \rightarrow f_2 - f_3 = -10$

C: $f_1 + f_3 = 30$

$$\begin{array}{ccc} f_1 & f_2 & f_3 \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 20 \\ 0 & 1 & -1 & -10 \\ 1 & 0 & 1 & 30 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 20 \\ 0 & 1 & -1 & -10 \\ 0 & -1 & 1 & 10 \end{array}\right]$$

$R_1 - R_3$

$R_1 - R_2$
$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 30 \\ 0 & 1 & -1 & -10 \\ 0 & 0 & 0 & 0 \end{array}\right]$$
$R_3 + R_2$

$\uparrow$

$f_3 = t$ $(t \geq 0)$

$f_1 + f_3 = 30 \Rightarrow f_1 = 30 - t$ $(t \leq 30)$

$f_2 - f_3 = -10 \Rightarrow f_2 = -10 + t$ $(t \geq 10)$

The intersection of the t-values is $10 \leq t \leq 30$

$[f_1, f_2, f_3] = [30-t, t-10, t]$ for $10 \leq t \leq 30$

b) If $f_2 = 5$ then $\boxed{t - 10 = 5 \Rightarrow t = 15}$ and $f_3 = 15$

$\rightarrow$

(15) Cont'd

c) Given $10 \le t \le 30$ from part a).

Sub $t = 10$ and $t = 30$ into:

$$\begin{cases} f_1 = 30 - t \\ f_2 = t - 10 \\ f_3 = t \end{cases}$$

$$\boxed{0 \le f_1 \le 20}$$

$$\boxed{0 \le f_2 \le 20}$$

$$\boxed{10 \le f_3 \le 30}$$

d) Negative flow would mean water flowing backwards.

(45) a) $y = ax^2 + bx + c$

$x=0, y=1: \quad 1 = c$

$x=-1, y=4: \quad 4 = a - b + c$

$x=2, y=1: \quad 1 = 4a + 2b + c$

$$\begin{array}{ccc|c} a & b & c & \\ \end{array}$$

$$\left[\begin{array}{ccc|c} 0 & 0 & 1 & 1 \\ 1 & -1 & 1 & 4 \\ 4 & 2 & 1 & 1 \end{array}\right]$$

$R_1 \leftrightarrow R_2$
$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 4 \\ 0 & 0 & 1 & 1 \\ 4 & 2 & 1 & 1 \end{array}\right]$$

$R_3 - 4R_1$
$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 4 \\ 0 & 0 & 1 & 1 \\ 0 & 6 & -3 & -15 \end{array}\right]$$

$\frac{R_3}{6}$ then $R_2 \leftrightarrow R_3$
$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 4 \\ 0 & 1 & -\frac{1}{2} & -\frac{5}{2} \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$R_1 + R_2$
$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & \frac{3}{2} \\ 0 & 1 & -\frac{1}{2} & -\frac{5}{2} \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$R_1 - \frac{1}{2}R_3$
$R_2 + \frac{1}{2}R_3$
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$[a, b, c] = [1, -2, 1]$

$y = x^2 - 2x + 1$