

⑤

Let $x =$ # bags of house blend $y =$ " special " $z =$ " gourmet "

$$\begin{array}{lcl}
 \text{kg of Columbian beans :} & 0.3x + 0.2y + 0.1z = 30 \\
 \text{" Kenyan "} & 0.2y + 0.2z = 15 \\
 \text{" French "} & 0.2x + 0.1y + 0.2z = 25
 \end{array}$$

$$\begin{array}{ccc|c}
 x & y & z & \\
 \hline
 0.3 & 0.2 & 0.1 & 30 \\
 0 & 0.2 & 0.2 & 15 \\
 0.2 & 0.1 & 0.2 & 25
 \end{array}$$

$$R_1 \leftrightarrow R_3 \quad \begin{array}{ccc|c}
 0.2 & 0.1 & 0.2 & 25 \\
 0 & 0.2 & 0.2 & 15 \\
 0.3 & 0.2 & 0.1 & 30
 \end{array}$$

$$5R_1 \quad \begin{array}{ccc|c}
 1 & 0.5 & 1 & 125 \\
 0 & 0.2 & 0.2 & 15 \\
 0.3 & 0.2 & 0.1 & 30
 \end{array}$$

$$R_3 - 0.3R_1 \quad \begin{array}{ccc|c}
 1 & 0.5 & 1 & 125 \\
 0 & 0.2 & 0.2 & 15 \\
 0 & 0.05 & -0.2 & -7.5
 \end{array}$$

$$5R_2 \quad \begin{array}{ccc|c}
 1 & 0.5 & 1 & 125 \\
 0 & 1 & 1 & 75 \\
 0 & 0.05 & -0.2 & -7.5
 \end{array}$$

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Recopying matrix:

$$\left[\begin{array}{ccc|c} 1 & 0.5 & 1 & 125 \\ 0 & 1 & 1 & 75 \\ 0 & 0.05 & -0.2 & -7.5 \end{array} \right]$$

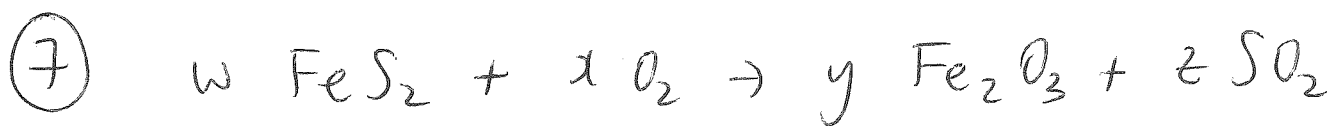
$$\begin{array}{l} R_1 - 0.5R_2 \\ R_3 - 0.05R_2 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0.5 & 87.5 \\ 0 & 1 & 1 & 75 \\ 0 & 0 & -0.25 & -11.25 \end{array} \right]$$

$$-4R_3 \left[\begin{array}{ccc|c} 1 & 0 & 0.5 & 87.5 \\ 0 & 1 & 1 & 75 \\ 0 & 0 & 1 & 45 \end{array} \right]$$

$$\begin{array}{l} R_1 - 0.5R_3 \\ R_2 - R_3 \end{array} \begin{array}{c} x \quad y \quad z \\ \left[\begin{array}{ccc|c} 1 & 0 & 0 & 65 \\ 0 & 1 & 0 & 30 \\ 0 & 0 & 1 & 45 \end{array} \right] \end{array}$$

The merchant can make:

65	bags of	house blend
30	"	special "
45	"	gourmet "



$$\text{Fe: } w = 2y \rightarrow \begin{array}{rcl} w & -2y & = 0 \end{array}$$

$$\text{S: } 2w = z \rightarrow \begin{array}{rcl} 2w & & -z = 0 \end{array}$$

$$\text{O: } 2x = 3y + 2z \rightarrow \begin{array}{rcl} 2x & -3y & -2z = 0 \end{array}$$

$$\begin{array}{cccc|c} w & x & y & z & \\ \hline 1 & 0 & -2 & 0 & 0 \\ 2 & 0 & 0 & -1 & 0 \\ 0 & 2 & -3 & -2 & 0 \end{array}$$

$$R_2 - 2R_1 \quad \begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & 0 & 4 & -1 & 0 \\ 0 & 2 & -3 & -2 & 0 \end{array}$$

$$\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & -3/2 & -1 & 0 \\ 0 & 0 & 4 & -1 & 0 \end{array}$$

$\frac{R_3}{2}$
then $R_2 \leftrightarrow R_3$

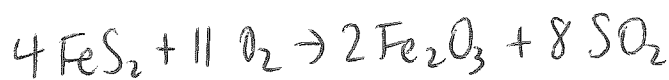
$$\frac{R_3}{4} \quad \begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & -3/2 & -1 & 0 \\ 0 & 0 & 1 & -1/4 & 0 \end{array}$$

$$\begin{array}{l} R_1 + 2R_3 \\ R_2 + \frac{3}{2}R_3 \end{array} \quad \begin{array}{cccc|c} 1 & 0 & 0 & -1/2 & 0 \\ 0 & 1 & 0 & -1/8 & 0 \\ 0 & 0 & 1 & -1/4 & 0 \end{array}$$

$$\left\{ \begin{array}{l} z = t \\ w = \frac{t}{2} \\ x = \frac{11t}{8} \\ y = \frac{t}{4} \end{array} \right.$$

Want smallest non-negative integer solution.

$$t=8 \Rightarrow [w, x, y, z] = [4, 11, 2, 8]$$



(15) a) Inflow = Outflow

A: $20 = f_1 + f_2 \rightarrow f_1 + f_2 = 20$

B: $10 + f_2 = f_3 \rightarrow f_2 - f_3 = -10$

C: $f_1 + f_3 = 30$

$$\begin{array}{ccc|c} f_1 & f_2 & f_3 & \\ \hline 1 & 1 & 0 & 20 \\ 0 & 1 & -1 & -10 \\ 1 & 0 & 1 & 30 \end{array}$$

$R_1 - R_3$

$$\begin{array}{ccc|c} 1 & 1 & 0 & 20 \\ 0 & 1 & -1 & -10 \\ 0 & -1 & 1 & 10 \end{array}$$

$R_1 - R_2$

$$\begin{array}{ccc|c} 1 & 0 & 1 & 30 \\ 0 & 1 & -1 & -10 \\ 0 & 0 & 0 & 0 \end{array}$$

$R_3 + R_2$

$f_3 = t$

$(t \geq 0)$

$(t \leq 30)$

$f_1 + f_3 = 30 \Rightarrow f_1 = 30 - t$

$f_2 - f_3 = -10 \Rightarrow f_2 = -10 + t$

$(t \geq 10)$

The intersection of the t -values is $10 \leq t \leq 30$

$[f_1, f_2, f_3] = [30 - t, t - 10, t]$ for $10 \leq t \leq 30$

b) If $f_2 = 5$ then $f_3 = 15$

$$\begin{array}{l} t - 10 = 5 \\ \Rightarrow t = 15 \end{array}$$

\rightarrow

(15) Cont'd
c)

Given $10 \leq t \leq 30$ from part a).

Sub $t=10$ and $t=30$ into :

$$\begin{cases} f_1 = 30 - t \\ f_2 = t - 10 \\ f_3 = t \end{cases}$$

$$0 \leq f_1 \leq 20$$

$$0 \leq f_2 \leq 20$$

$$10 \leq f_3 \leq 30$$

d) Negative flow would mean water flowing backwards.

(45) a)

$$y = ax^2 + bx + c$$

$$x=0, y=1: \quad 1 = c$$

$$x=-1, y=4: \quad 4 = a - b + c$$

$$x=2, y=1: \quad 1 = 4a + 2b + c$$

$$\begin{bmatrix} a & b & c & | & \\ 0 & 0 & 1 & | & 1 \\ 1 & -1 & 1 & | & 4 \\ 4 & 2 & 1 & | & 1 \end{bmatrix}$$

$$R_1 \leftrightarrow R_2 \quad \begin{bmatrix} 1 & -1 & 1 & | & 4 \\ 0 & 0 & 1 & | & 1 \\ 4 & 2 & 1 & | & 1 \end{bmatrix}$$

$$R_3 - 4R_1 \quad \begin{bmatrix} 1 & -1 & 1 & | & 4 \\ 0 & 0 & 1 & | & 1 \\ 0 & 6 & -3 & | & -15 \end{bmatrix}$$

$$\frac{R_3}{6} \text{ then } R_2 \leftrightarrow R_3 \quad \begin{bmatrix} 1 & -1 & 1 & | & 4 \\ 0 & 1 & \frac{1}{2} & | & \frac{1}{2} \\ 0 & 0 & 1 & | & 1 \end{bmatrix}$$

$$R_1 + R_2 \quad \begin{bmatrix} 1 & 0 & \frac{3}{2} & | & \frac{9}{2} \\ 0 & 1 & \frac{1}{2} & | & \frac{1}{2} \\ 0 & 0 & 1 & | & 1 \end{bmatrix}$$

$$\begin{array}{l} R_1 - \frac{3}{2}R_3 \\ R_2 + \frac{1}{2}R_3 \end{array} \quad \begin{bmatrix} 1 & 0 & 0 & | & 1 \\ 0 & 1 & 0 & | & -2 \\ 0 & 0 & 1 & | & 1 \end{bmatrix}$$
$$[a, b, c] = [1, -2, 1]$$

$$y = x^2 - 2x + 1$$