2.3

(1) Let $c_1\vec{u}_1 + c_2\vec{u}_2 = \vec{v}$

$$c_1\begin{bmatrix}1\\-1\end{bmatrix} + c_2\begin{bmatrix}2\\-1\end{bmatrix} = \begin{bmatrix}1\\2\end{bmatrix}$$

$$\begin{array}{ccc} c_1 & c_2 & \\ \end{array}$$
$$\left[\begin{array}{cc|c} 1 & 2 & 1 \\ -1 & -1 & 2 \end{array}\right]$$

$$R_2 + R_1 \quad \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & 1 & 3 \end{array}\right]$$

REF

System is consistent (solvable).

$\boxed{\text{Yes}}$

(3) Let $c_1\vec{u}_1 + c_2\vec{u}_2 = \vec{v}$

$$c_1\begin{bmatrix}1\\1\\0\end{bmatrix} + c_2\begin{bmatrix}0\\1\\1\end{bmatrix} = \begin{bmatrix}1\\2\\3\end{bmatrix}$$

$$\begin{array}{cc} c_1 & c_2 \end{array}$$
$$\left[\begin{array}{cc|c} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 0 & 1 & 3 \end{array}\right]$$

$$R_2 - R_1 \quad \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 3 \end{array}\right]$$

$$R_3 - R_2 \quad \left[\begin{array}{cc|c} & & \\ & & \\ 0 & 0 & 2 \end{array}\right]$$

System is inconsistent.
(System is not solvable).

$\boxed{\text{No}}$

⑤ Let $c_1\bar{u}_1 + c_2\bar{u}_2 + c_3\bar{u}_3 = \bar{v}$

$$c_1\begin{bmatrix}1\\1\\0\end{bmatrix} + c_2\begin{bmatrix}0\\1\\1\end{bmatrix} + c_3\begin{bmatrix}1\\0\\1\end{bmatrix} = \begin{bmatrix}1\\2\\3\end{bmatrix}$$

$$\begin{array}{ccc} c_1 & c_2 & c_3 \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 2 \\ 0 & 1 & 1 & 3 \end{array}\right]$$

$$R_2 - R_1 \quad \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 1 & 1 & 3 \end{array}\right]$$

$$R_3 - R_2 \quad \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 2 & 2 \end{array}\right]$$

REF

System is consistent.

$\boxed{\text{Yes}}$

(7) Is $\bar{b} = c_1\begin{bmatrix}1\\3\end{bmatrix} + c_2\begin{bmatrix}2\\4\end{bmatrix}$?

Let $\bar{b} = c_1\begin{bmatrix}1\\3\end{bmatrix} + c_2\begin{bmatrix}2\\4\end{bmatrix}$

$$\begin{array}{cc} c_1 & c_2 \end{array}$$
$$\left[\begin{array}{cc|c} 1 & 2 & 5 \\ 3 & 4 & 6 \end{array}\right]$$

$$R_2-3R_1 \quad \left[\begin{array}{cc|c} 1 & 2 & 5 \\ 0 & -2 & -9 \end{array}\right]$$

REF

System is consistent.

$\boxed{\text{Yes}}$

(11) Let $c_1\begin{bmatrix}1\\0\\1\end{bmatrix} + c_2\begin{bmatrix}1\\1\\0\end{bmatrix} + c_3\begin{bmatrix}0\\1\\1\end{bmatrix} = \begin{bmatrix}a\\b\\c\end{bmatrix}$

Show that the system is consistent.

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & a \\ 0 & 1 & 1 & b \\ 1 & 0 & 1 & c \end{array}\right]$$

$$R_3-R_1 \quad \left[\begin{array}{ccc|c} 1 & 1 & 0 & a \\ 0 & 1 & 1 & b \\ 0 & -1 & 1 & c-a \end{array}\right]$$

$$R_3+R_2 \quad \left[\begin{array}{ccc|c} 1 & 1 & 0 & a \\ 0 & 1 & 1 & b \\ 0 & 0 & 2 & b+c-a \end{array}\right] \quad \text{REF}$$

System is consistent.

(13) Notice that the vectors are parallel.

$$\text{span}\left(\begin{bmatrix}2\\-4\end{bmatrix}, \begin{bmatrix}-1\\2\end{bmatrix}\right)$$

$$= \text{span}\left(\begin{bmatrix}-1\\2\end{bmatrix}\right)$$

a) Geometric Description:

Line through the origin with direction vector $\vec{d} = \begin{bmatrix}-1\\2\end{bmatrix}$

b) Algebraic Description (4 possibilities)

$$\bar{x} = t\begin{bmatrix}-1\\2\end{bmatrix}$$

or $\begin{cases} x = -t \\ y = 2t \end{cases}$

or $\vec{n}$ is $\perp$ to $\begin{bmatrix}-1\\2\end{bmatrix}$

$\Rightarrow \vec{n} = \begin{bmatrix}2\\1\end{bmatrix}$

$$\vec{n} \cdot \bar{x} = \vec{n} \cdot \vec{p}$$

$$\begin{bmatrix}2\\1\end{bmatrix} \cdot \begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}2\\1\end{bmatrix} \cdot \begin{bmatrix}0\\0\end{bmatrix}$$

or $2x + y = 0$

(15) Notice that vectors are <u>not</u> parallel

a) Geometric Description:

Plane through the origin with direction vectors $\begin{bmatrix}1\\2\\0\end{bmatrix}$ and $\begin{bmatrix}3\\2\\-1\end{bmatrix}$

b) Algebraic Description (4 possibilities)

$$\bar{x} = s\begin{bmatrix}1\\2\\0\end{bmatrix} + t\begin{bmatrix}3\\2\\-1\end{bmatrix}$$

or $$\begin{cases} x = s + 3t \\ y = 2s + 2t \\ z = -t \end{cases}$$

or $$\bar{n} = \begin{bmatrix}1\\2\\0\end{bmatrix} \times \begin{bmatrix}3\\2\\-1\end{bmatrix}$$

$$= \begin{bmatrix}-2\\1\\-4\end{bmatrix}$$

$$\boxed{\begin{array}{ccccc}1&2&0&1&2\\3&2&-1&3&2\end{array}}$$

$$\bar{n}\cdot\bar{x} = \bar{n}\cdot\bar{p}$$

$$\begin{bmatrix}-2\\1\\-4\end{bmatrix}\cdot\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}-2\\1\\-4\end{bmatrix}\cdot\begin{bmatrix}0\\0\\0\end{bmatrix}$$

or $$-2x + y - 4z = 0$$

$$2x - y + 4z = 0$$

(23) Let $c_1\begin{bmatrix}1\\1\\1\end{bmatrix}+c_2\begin{bmatrix}1\\2\\3\end{bmatrix}+c_3\begin{bmatrix}1\\-1\\2\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$

$$\begin{array}{ccc} c_1 & c_2 & c_3 \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & 2 & -1 & 0 \\ 1 & 3 & 2 & 0 \end{array}\right]$$

$$\begin{array}{l} \\ R_2-R_1 \\ R_3-R_1 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 2 & 1 & 0 \end{array}\right]$$

$$\begin{array}{l} R_1-R_2 \\ \\ R_3-2R_2 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 5 & 0 \end{array}\right]$$

$$\begin{array}{l} \\ \\ \frac{R_3}{5} \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\begin{array}{l} R_1-3R_3 \\ R_2+2R_3 \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$c_1 = c_2 = c_3 = 0$$

The vectors are linearly independent.

(27) Let $c_1\begin{bmatrix}3\\4\\5\end{bmatrix}+c_2\begin{bmatrix}6\\7\\8\end{bmatrix}+c_3\begin{bmatrix}0\\0\\0\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$

By inspection the vectors are linearly dependent because $\begin{bmatrix}0\\0\\0\end{bmatrix}$ is one of the vectors.

A dependence relationship:

$$0\begin{bmatrix}3\\4\\5\end{bmatrix}+0\begin{bmatrix}6\\7\\8\end{bmatrix}+c\begin{bmatrix}0\\0\\0\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

where c is any <u>nonzero</u> real number.

(29) Let $c_1\begin{bmatrix}1\\-1\\1\\0\end{bmatrix}+c_2\begin{bmatrix}-1\\1\\0\\1\end{bmatrix}+c_3\begin{bmatrix}1\\0\\1\\-1\end{bmatrix}+c_4\begin{bmatrix}0\\1\\-1\\1\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$

$$\begin{array}{c} \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \\ \left[\begin{array}{cccc|c} 1 & -1 & 1 & 0 & 0\\ -1 & 1 & 0 & 1 & 0\\ 1 & 0 & 1 & -1 & 0\\ 0 & 1 & -1 & 1 & 0 \end{array}\right] \end{array}$$

$$\begin{array}{c} R_2+R_1 \\ R_3-R_1 \end{array} \left[\begin{array}{cccc|c} 1 & -1 & 1 & 0 & 0\\ 0 & 0 & 1 & 1 & 0\\ 0 & 1 & 0 & -1 & 0\\ 0 & 1 & -1 & 1 & 0 \end{array}\right]$$

$$R_2 \leftrightarrow R_3 \left[\begin{array}{cccc|c} 1 & -1 & 1 & 0 & 0\\ 0 & 1 & 0 & -1 & 0\\ 0 & 0 & 1 & 1 & 0\\ 0 & 1 & -1 & 1 & 0 \end{array}\right] \rightarrow$$

Recopying previous matrix:

$$\left[\begin{array}{cccc|c} 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 1 & 0 \end{array}\right]$$

$$\begin{array}{c} \\ \\ \\ R_4 - R_2 \end{array} \left[\begin{array}{cccc|c} 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & 2 & 0 \end{array}\right]$$

$$\begin{array}{c} \\ \\ \\ R_4 + R_3 \end{array} \left[\begin{array}{cccc|c} 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 3 & 0 \end{array}\right] \quad \text{REF}$$

By back-substitution

the solution is $C_1 = C_2 = C_3 = C_4 = 0$.

The vectors are linearly independent.