Part I

Cross Product
Part I

1) a)

$$\begin{array}{ccccc} 0 & 1 & 1 & 0 & 1 \\ & \times & \times & \times & \\ 3 & -1 & 2 & 3 & -1 \end{array}$$

$\vec{a} \times \vec{b} = [3, 3, -3]$

b)

$$\begin{array}{ccccc} 1 & 1 & 1 & 1 & 1 \\ & \times & \times & \times & \\ 1 & 2 & 3 & 1 & 2 \end{array}$$

$\vec{a} \times \vec{b} = [1, -2, 1]$

2) a) $\vec{u} \times \vec{v}$ is a normal for the plane containing $\vec{u}$ and $\vec{v}$

$$\begin{array}{ccccc} 0 & 1 & 2 & 0 & 1 \\ & \times & \times & \times & \\ 1 & 1 & 4 & 1 & 1 \end{array}$$

$\vec{n} = [2, 2, -1]$
or any nonzero multiple of $[2, 2, -1]$

b) Let $\bar{u} = \overrightarrow{PQ} = [2, 1, 1]$

$\bar{v} = \overrightarrow{PR} = [1, 3, -2]$

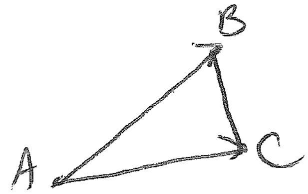
$$\begin{array}{ccccccccc} 2 & & 1 & & 1 & & 2 & & 1 \\ & & & \times & & \times & & \times & \\ 1 & & 3 & & -2 & & 1 & & 3 \end{array}$$

$\bar{n} = [-5, 5, 5]$

or any nonzero multiple of $[-5, 5, 5]$

(3) Let $\bar{u} = \overrightarrow{AB}$

$\bar{v} = \overrightarrow{AC}$



$\bar{u} = [1, -1, -1]$

$\bar{v} = [4, -3, 2]$

$$\begin{array}{ccccccccc} 1 & & -1 & & -1 & & 1 & & -1 \\ & & & \times & & \times & & \times & \\ 4 & & -3 & & 2 & & 4 & & -3 \end{array}$$

$\bar{u} \times \bar{v} = [-5, -6, 1]$

Area of any triangle ABC is $\frac{1}{2}\|\bar{u} \times \bar{v}\|$, where $\bar{u}$ and $\bar{v}$ are two of the sides.

Area $= \frac{\sqrt{62}}{2}$

Part II

Cross Product Part II

① $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & 1 \\ 3 & -1 & 2 \end{vmatrix}$

$= \vec{i}(3) - \vec{j}(-3) + \vec{k}(-3)$

$= [3, 3, -3]$

② V (parallelepiped)

= absolute value of $\det \begin{bmatrix} \vec{u} \\ \vec{v} \\ \vec{w} \end{bmatrix}$

$= \left| \begin{vmatrix} 1 & 4 & 9 \\ 2 & -6 & 3 \\ -1 & 1 & 1 \end{vmatrix} \right|$

$= | 1(-9) - 4(5) + 9(-4) |$

$= |-65|$

$= 65$

(3) The three vectors lie in a plane if and only if $V(\text{parallelepiped}) = 0$

$V(\text{parallelepiped})$

$$= \left| \begin{vmatrix} 1 & 7 & -2 \\ 2 & 3 & -2 \\ -2 & 5 & -2 \end{vmatrix} \right|$$

$$= | 1(4) - 7(-8) - 2(16) |$$

$$= | 28 |$$

$$= 28$$

No, the three vectors do not lie in a plane.

(4) Area of parallelogram in $\mathbb{R}^2$
= absolute value of $\det \begin{bmatrix} \vec{u} \\ \vec{v} \end{bmatrix}$

$$= \left| \begin{vmatrix} -2 & 3 \\ 4 & 9 \end{vmatrix} \right|$$

$$= | -30 |$$

$$= 30$$