(1) Normal form

$$\vec{n}\cdot\vec{x} = \vec{n}\cdot\vec{p}$$

$$\begin{bmatrix}3\\2\end{bmatrix}\cdot\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}3\\2\end{bmatrix}\cdot\begin{bmatrix}0\\0\end{bmatrix}$$

General form

$$3x + 2y = 0$$

(3) Vector form

$$\vec{x} = \vec{p} + t\vec{d}$$

$$\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix} + t\begin{bmatrix}-1\\3\end{bmatrix}$$

Parametric form

$$\begin{cases} x = 1 - t \\ y = 3t \end{cases}$$

(5) Vector form

$$\vec{x} = \vec{p} + t\vec{d}$$

$$\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix} + t\begin{bmatrix}1\\-1\\4\end{bmatrix}$$

Parametric form

$$\begin{cases} x = t \\ y = -t \\ z = 4t \end{cases}$$

(7) Normal form

$$\vec{n}\cdot\vec{x} = \vec{n}\cdot\vec{p}$$

$$\begin{bmatrix}3\\2\\1\end{bmatrix}\cdot\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}3\\2\\1\end{bmatrix}\cdot\begin{bmatrix}0\\1\\0\end{bmatrix}$$

General form

$$3x + 2y + z = 2$$

(9) Vector form $\vec{x} = \vec{p} + s\vec{u} + t\vec{v}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} + t\begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}$$

Parametric form
$$\begin{cases} x = 2s - 3t \\ y = s + 2t \\ z = 2s + t \end{cases}$$

(13) Let $\vec{u} = \vec{PQ} = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$

$\vec{v} = \vec{PR} = \begin{bmatrix} -1 \\ 0 \\ -2 \end{bmatrix}$

$\vec{x} = \vec{p} + s\vec{u} + t\vec{v}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + s\begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix} + t\begin{bmatrix} -1 \\ 0 \\ -2 \end{bmatrix}$$

(15) a) Find two points on the line
$P = (0, -1)$ and $Q = (1, 2)$

$\vec{d} = \vec{PQ} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

Vector form $\vec{x} = \vec{p} + t\vec{d}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} + t\begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

Parametric form
$$\begin{cases} x = t \\ y = -1 + 3t \end{cases}$$

$\rightarrow$

(15) b) Find two points on the line

$P = \left(\frac{5}{3}, 0\right)$ and $Q = \left(0, \frac{5}{2}\right)$

$\vec{d} = \overrightarrow{PQ} = \begin{bmatrix} -\frac{5}{3} \\ \frac{5}{2} \end{bmatrix}$

Vector form $\quad \bar{x} = \bar{p} + t\vec{d}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{5}{3} \\ 0 \end{bmatrix} + t\begin{bmatrix} -\frac{5}{3} \\ \frac{5}{2} \end{bmatrix}$$

Parametric form
$$\begin{cases} x = \frac{5}{3} - \frac{5}{3}t \\ y = \frac{5}{2}t \end{cases}$$

(19) Check if the normals are parallel or perpendicular to the normal of P_1, which is $[4, -1, 5]$

a) $[2, 3, -1] \cdot [4, -1, 5] = 0$
Normals are perpendicular
$\Rightarrow$ Planes are perpendicular

b) $[4, -1, 5]$ is a multiple of $[4, -1, 5]$
Normals are parallel
$\Rightarrow$ Planes are parallel

$\rightarrow$

(19) c) $[1,-1,-1]\cdot[4,-1,+5]=0$

Normals are perpendicular

$\Rightarrow$ Planes are perpendicular

d) $[4,6,-2]\cdot[4,-1,5]=0$

Normals are perpendicular

$\Rightarrow$ Planes are perpendicular

(21) Let $\mathcal{L}$: $2x-3y=1$

Direction vector for $\mathcal{L}$ is a direction vector for the desired line.

Two points on $\mathcal{L}$: $P=(\frac{1}{2},0)$ and $Q=(0,-\frac{1}{3})$

$$\vec{d}=\vec{PQ}=[-\tfrac{1}{2},-\tfrac{1}{3}]$$

Vector form of desired line: $\vec{x}=\vec{p}+t\vec{d}$

$$\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}2\\-1\end{bmatrix}+t\begin{bmatrix}-\frac{1}{2}\\-\frac{1}{3}\end{bmatrix}$$

Can multiply direction vector by any nonzero #

Alternatively: $\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}2\\-1\end{bmatrix}+t\begin{bmatrix}3\\2\end{bmatrix}$

etc.

Section 1.3

(23) Let $\mathcal{L}$: $\begin{cases} x = 1 - t \\ y = 2 + 3t \\ z = -2 - t \end{cases}$

$\mathcal{L}$: $\bar{x} = \bar{p} + t\begin{bmatrix} -1 \\ 3 \\ -1 \end{bmatrix}$

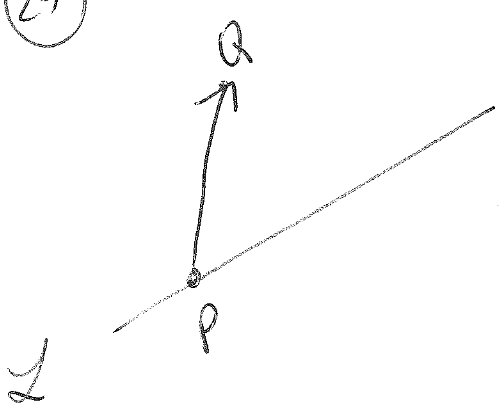
$\mathcal{L}$ has direction vector $\bar{d} = \begin{bmatrix} -1 \\ 3 \\ -1 \end{bmatrix}$

Desired line has same direction vector

Desired line in vector form: $\bar{x} = \bar{p} + t\bar{d}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 3 \end{bmatrix} + t\begin{bmatrix} -1 \\ 3 \\ -1 \end{bmatrix}$$

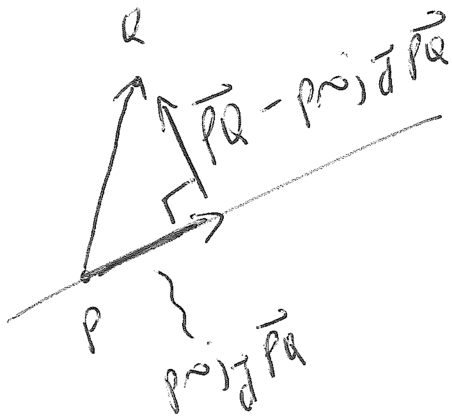
(27)



Let P be any point on $\mathcal{L}$

$P = (-1, 2)$

$\vec{PQ} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$



$\rightarrow$

27 Cont'd

$$\vec{d} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\text{proj}_{\vec{d}}\, \overrightarrow{PQ} = \frac{\vec{d} \cdot \overrightarrow{PQ}}{\|\vec{d}\|^2}\, \vec{d}$$

$$= \frac{3}{2} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\overrightarrow{PQ} - \text{proj}_{\vec{d}}\, \overrightarrow{PQ} = \begin{bmatrix} 3 \\ 0 \end{bmatrix} - \frac{3}{2} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

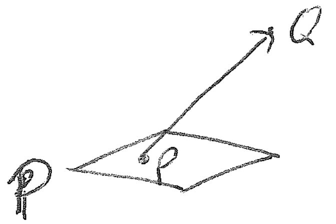
$$= \begin{bmatrix} \frac{3}{2} \\ \frac{3}{2} \end{bmatrix}$$

$$= \frac{3}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\text{Distance} = \| \overrightarrow{PQ} - \text{proj}_{\vec{d}}\, \overrightarrow{PQ} \|$$

$$= \frac{3}{2}\sqrt{2} \quad \text{or} \quad \frac{3\sqrt{2}}{2}$$

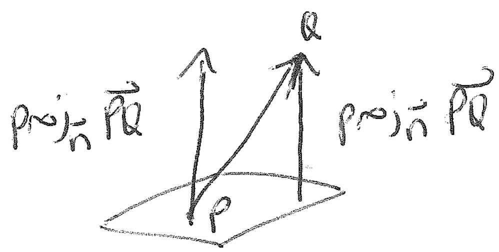
(29)



Let P be any point on $\mathbb{P}$

$P = (0,0,0)$

$$\vec{PQ} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$



$$\vec{n} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

$$\text{proj}_{\vec{n}} \vec{PQ} = \frac{\vec{n} \cdot \vec{PQ}}{\|\vec{n}\|^2} \vec{n}$$

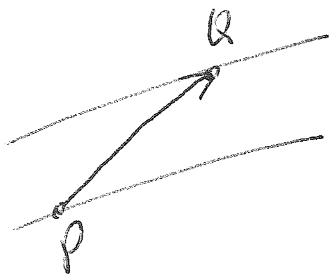
$$= \frac{2}{3} \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

$$\text{Distance} = \|\text{proj}_{\vec{n}} \vec{PQ}\|$$

$$= \frac{2}{3}\sqrt{3} \quad \text{or} \quad \frac{2\sqrt{3}}{3}$$

(35)

Section 1.3

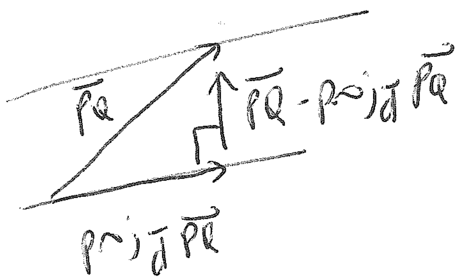


Choose points P and Q on each line

$P = (1,1)$

$Q = (5,4)$

$$\overline{PQ} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$



$$\vec{d} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

$$\text{proj}_{\vec{d}} \overline{PQ} = \frac{\vec{d} \cdot \overline{PQ}}{\|\vec{d}\|^2} \vec{d}$$

$$= \frac{1}{13} \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

$$\overline{PQ} - \text{proj}_{\vec{d}} \overline{PQ} = \begin{bmatrix} 4 \\ 3 \end{bmatrix} - \frac{1}{13} \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

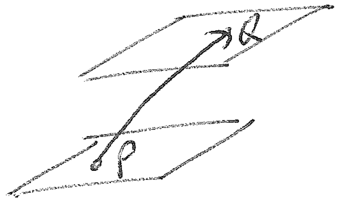
$$= \begin{bmatrix} \frac{54}{13} \\ \frac{36}{13} \end{bmatrix}$$

$$= \frac{1}{13} \begin{bmatrix} 54 \\ 36 \end{bmatrix}$$

$$\text{Distance} = \| \overline{PQ} - \text{proj}_{\vec{d}} \overline{PQ} \|$$

$$= \frac{1}{13} (18\sqrt{13}) \quad \text{or} \quad \frac{18\sqrt{13}}{13}$$

(37)

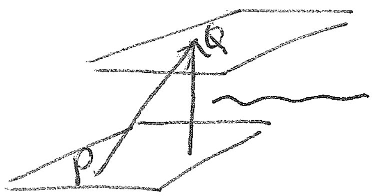


Choose points P and Q on each plane

$P = (0,0,0)$

$Q = (0,5,0)$

$\overline{PQ} = \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix}$



$\text{proj}_{\vec{n}} \overline{PQ}$

$\vec{n} = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$

$$\text{proj}_{\vec{n}} \overline{PQ} = \frac{\vec{n} \cdot \overline{PQ}}{\|\vec{n}\|^2} \vec{n}$$

$$= \frac{5}{9} \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$$

$$\text{Distance} = \|\text{proj}_{\vec{n}} \overline{PQ}\|$$

$$= \frac{5}{9}\sqrt{9}$$

$$= \frac{5}{3}$$

43 Find the angle between the two normals

Section 1.3

$\vec{n}_1 = [1, 1, 1]$

$\vec{n}_2 = [2, 1, -2]$

$$\vec{n}_1 \cdot \vec{n}_2 = \|\vec{n}_1\| \|\vec{n}_2\| \cos\theta$$

$$1 = \sqrt{3}\sqrt{9}\cos\theta$$

$$\frac{1}{3\sqrt{3}} = \cos\theta$$

$$\theta = \cos^{-1}\left(\frac{1}{3\sqrt{3}}\right)$$

$$\theta \simeq 78.9^\circ$$