Section 1.2

(3) $\vec{u} \cdot \vec{v} = 1(2) + 2(3) + 3(1) = 11$

(5) $\vec{u} \cdot \vec{v} = 1(4) + \sqrt{2}(-\sqrt{2}) + \sqrt{3}(0) + 0(-5)$

$= 2$

(11) $\|\vec{u}\| = \sqrt{1^2 + \sqrt{2}^2 + \sqrt{3}^2 + 0^2}$

$= \sqrt{6}$

$$\frac{\vec{u}}{\|\vec{u}\|} = \frac{1}{\sqrt{6}}[1, \sqrt{2}, \sqrt{3}, 0]$$

or $\left[\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{2}}, 0\right]$

(15) $d(\vec{u}, \vec{v}) = \|\vec{u} - \vec{v}\|$

$$= \left\| \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix} \right\|$$

$= \sqrt{6}$

(17) a) $\vec{u} \cdot \vec{v}$ is a scalar
Cannot calculate length of a scalar

b) $\vec{u} \cdot \vec{v}$ is a scalar
Cannot add a scalar and a vector

c) $\vec{v} \cdot \vec{w}$ is a scalar
Cannot calculate dot product of a vector and a scalar

d) c is a scalar
Cannot calculate dot product of a scalar and a vector

(19) $\bar{u} \cdot \bar{v} = 2(1) + (-1)(-2) + 1(-1)$

$= 3$

$\bar{u} \cdot \bar{v} > 0 \Rightarrow$ the angle is acute

(25) $\bar{u} \cdot \bar{v} = \|\bar{u}\| \, \|\bar{v}\| \cos\theta$

$3 = \sqrt{6}\sqrt{6}\cos\theta$

$\cos\theta = \frac{1}{2}$

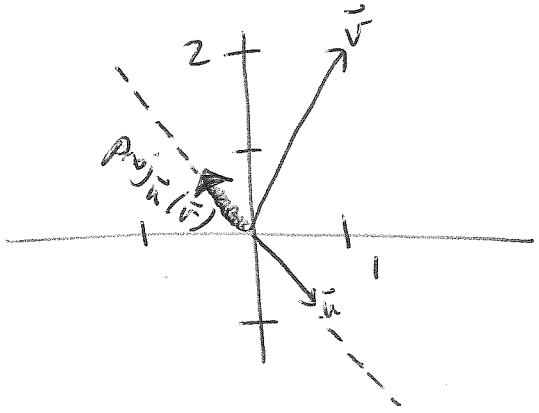
$\theta = \cos^{-1}\left(\frac{1}{2}\right)$

$\theta = 60°$

(41) $\text{proj}_{\bar{u}}(\bar{v}) = \frac{\bar{u} \cdot \bar{v}}{\|\bar{u}\|^2} \bar{u}$

$= \frac{-1}{1} \begin{bmatrix} 3/5 \\ -4/5 \end{bmatrix}$

$= \begin{bmatrix} -3/5 \\ 4/5 \end{bmatrix}$

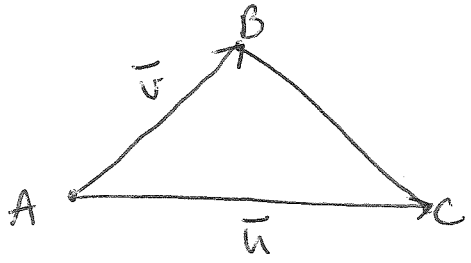


(43) $\text{proj}_{\bar{u}}(\bar{v}) = \dfrac{\bar{u}\cdot\bar{v}}{\|\bar{u}\|^2}\,\bar{u}$

$= \dfrac{6}{4}\begin{bmatrix}1\\-1\\1\\-1\end{bmatrix}$

$= \dfrac{1}{2}\begin{bmatrix}3\\-3\\3\\-3\end{bmatrix}$

(47) Compute both $\frac{1}{2}\|\bar{u}\|\,\|\bar{v}-\text{proj}_{\bar{u}}(\bar{v})\|$ and $\frac{1}{2}\|\bar{u}\|\|\bar{v}\|\sin\theta$



$\bar{u} = \overrightarrow{AC} = [2, 1, -2]$

$\bar{v} = \overrightarrow{AB} = [1, -1, 2]$

FIRST COMPUTATION:

$\|\bar{u}\| = \sqrt{9} = 3$

$\text{proj}_{\bar{u}}(\bar{v}) = \dfrac{\bar{u}\cdot\bar{v}}{\|\bar{u}\|^2}\,\bar{u}$

$= \dfrac{-3}{9}\,\bar{u}$

$= -\frac{1}{3}[2, 1, -2]$

$$\bar{v} - \text{proj}_{\bar{u}}(\bar{v}) = [1,-1,2] + \frac{1}{3}[2,1,-2]$$

$$= \left[\frac{5}{3}, \frac{-2}{3}, \frac{4}{3}\right]$$

$$= \frac{1}{3}[5,-2,4]$$

$$\|\bar{v} - \text{proj}_{\bar{u}}(\bar{v})\| = \frac{1}{3}\sqrt{45}$$

So $\frac{1}{2}\|\bar{u}\|\,\|\bar{v} - \text{proj}_{\bar{u}}(\bar{v})\| = \frac{1}{2}(3)\left(\frac{1}{3}\sqrt{45}\right)$

$$= \frac{\sqrt{45}}{2}$$

SECOND COMPUTATION:

$$\|\bar{v}\| = \sqrt{6}$$

$$\bar{u}\cdot\bar{v} = \|\bar{u}\|\,\|\bar{v}\|\cos\theta$$

$$-3 = 3\sqrt{6}\cos\theta$$

$$\cos\theta = \frac{-1}{\sqrt{6}}$$

$$\sin\theta = \sqrt{1-\cos^2\theta}$$

$$= \sqrt{\frac{5}{6}}$$

So $\frac{1}{2}\|\bar{u}\|\,\|\bar{v}\|\sin\theta = \frac{1}{2}(3)(\sqrt{6})\left(\sqrt{\frac{5}{6}}\right)$

$$= \frac{3}{2}\sqrt{5}$$

$$= \frac{\sqrt{45}}{2}$$

(49) $\bar{u}$ and $\bar{v}$ are orthogonal

$\Rightarrow \bar{u} \cdot \bar{v} = 0$

$k^2 - k - 6 = 0$

$(k-3)(k+2) = 0$

$k = 3, -2$

(63) $\frac{1}{4}\|\bar{u}+\bar{v}\|^2 - \frac{1}{4}\|\bar{u}-\bar{v}\|^2$

$= \frac{1}{4}(\bar{u}+\bar{v}) \cdot (\bar{u}+\bar{v}) - \frac{1}{4}(\bar{u}-\bar{v}) \cdot (\bar{u}-\bar{v})$

$= \frac{1}{4}[\bar{u} \cdot \bar{u} + 2(\bar{u} \cdot \bar{v}) + \bar{v} \cdot \bar{v}] - \frac{1}{4}[\bar{u} \cdot \bar{u} - 2(\bar{u} \cdot \bar{v}) + \bar{v} \cdot \bar{v}]$

$= \frac{1}{2}(\bar{u} \cdot \bar{v}) + \frac{1}{2}(\bar{u} \cdot \bar{v})$

$= \bar{u} \cdot \bar{v}$