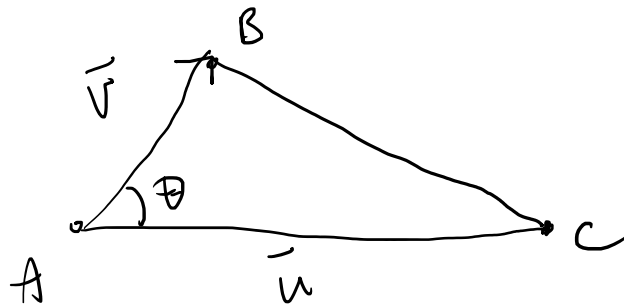


1.2 #47

1) Calculate  $\frac{1}{2} \|\vec{u}\| \|\vec{v}\| \sin \theta$ 

$$\vec{u} = \vec{AC} = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$$

$$\vec{v} = \vec{AB} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

$$\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos \theta$$

$$-3 = \sqrt{9} \sqrt{6} \cos \theta$$

$$\cos \theta = \frac{-1}{\sqrt{6}}$$

$$\sin \theta = \sqrt{1 - \cos^2 \theta}$$

$$= \sqrt{1 - \frac{1}{6}}$$

$$= \sqrt{\frac{5}{6}}$$

$$= \frac{\sqrt{5}}{\sqrt{6}}$$

$$\begin{aligned}
 \text{Area} &= \frac{1}{2} \|\vec{u}\| \|\vec{v}\| \sin \theta \\
 &= \frac{1}{2} \sqrt{9} \sqrt{6} \frac{\sqrt{5}}{\sqrt{6}} \\
 &= \frac{\sqrt{45}}{2}
 \end{aligned}$$

2) Calculate  $\frac{1}{2} \|\vec{u}\| \|\vec{v} - \text{proj}_{\vec{u}} \vec{v}\|$

$$\vec{u} = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$$

$$\vec{v} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

$$\text{proj}_{\vec{u}} \vec{v} = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\|^2} \vec{u}$$

$$= \frac{-3}{9} \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$$

$$= -\frac{1}{3} \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$$

$$\vec{v} - \text{proj}_{\vec{u}} \vec{v} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} + \frac{1}{3} \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} 5/3 \\ -2/3 \\ 4/3 \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix}$$

$$\text{Area} = \frac{1}{2} \|\vec{u}\| \|\vec{v} - \text{proj}_{\vec{u}} \vec{v}\|$$

$$= \frac{1}{2} \sqrt{9} \left( \frac{1}{3} \sqrt{45} \right)$$

$$= \frac{\sqrt{45}}{2}$$