2.4 Applications of Linear Systems

1. Given the points $(1, -2)$, $(-1, 8)$, and $(2, -1)$, find the equation $y = ax^2 + bx + c$ that contains them all.

2. NASA is planning a mission to Mars, and their dieticians must design a food supplement that provides 170 mg of Vitamin A, 210 mg of Vitamin C, and 370 mg of calcium. The three possible ingredients have:

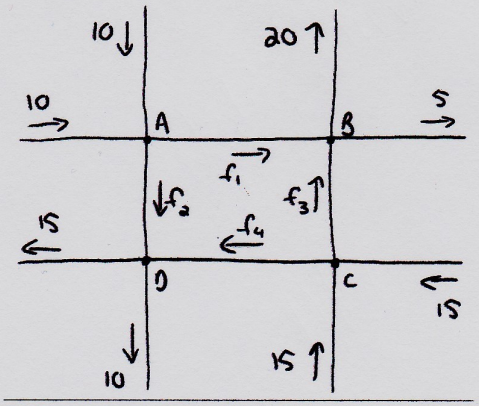
	ingredient 1	ingredient 2	ingredient 3
Vitamin A	10	30	20
Vitamin C	20	50	30
calcium	60	130	70

   where these numbers are milligrams per unit of food.

   Is it possible to design such a supplement from these ingredients? If so, how much of each ingredient must be used?

3. Balance $NH_3 + O_2 \rightarrow N_2 + H_2O$.

4. The downtown core of Gotham City consists of one-way streets, and the traffic flow has been measured at each intersection. For the city block show in the figure below, the numbers represent the average numbers of vehicles per minute entering and leaving intersections A, B, C, and D during business hours.



   (a) Set up and solve a system of linear equations to find the possible flows f_1 to f_4.

   (b) If traffic is regulated on CD such that $f_4 = 10$ vehicles per minute, what will the average flows on the other streets be?

   (c) What are the minimum and maximum possible flows on each street?

5. Find all possible combinations of 20 coins (nickels, dimes, and quarters) that will make exactly \$3.00.

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Ex 1: $y = ax^2 + bx + c$

$x = 1$, $y = -2$: $\quad -2 = a + b + c$

$x = -1$, $y = 8$: $\quad 8 = a - b + c$

$x = 2$, $y = -1$: $\quad -1 = 4a + 2b + c$

$$\left[\begin{array}{ccc|c} a & b & c & \\ 1 & 1 & 1 & -2 \\ 1 & -1 & 1 & 8 \\ 4 & 2 & 1 & -1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} a & b & c & \\ 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$a = 2$
$b = -5$
$c = 1$

$$\boxed{y = 2x^2 - 5x + 1}$$

Ex 2: x = # units of Ingredient 1
y = " 2
z = " 3

Vit A: $10x + 30y + 20z = 170$

Vit C: $20x + 50y + 30z = 210$

Ca: $60x + 130y + 70z = 370$

$$\begin{matrix} x & y & z & \# \end{matrix}$$
$$\left[\begin{array}{ccc|c} 10 & 30 & 20 & 170 \\ 20 & 50 & 30 & 210 \\ 60 & 130 & 70 & 370 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & -1 & -22 \\ 0 & 1 & 1 & 13 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$z = t$

$y = 13 - t$

$x = -22 + t$

Need $x, y, z \geq 0$?

$t \geq 0$

$t \leq 13$

$t \geq 22$

No, not possible.

Ex 3 :

$$w(NH_3) + x(O_2) = y(N_2) + z(H_2O)$$

N :	$w = 2y$	$w - 2y = 0$
H :	$3w = 2z$	$3w - 2z = 0$
O :	$2x = z$	$2x - z = 0$

$$\begin{array}{c} w \quad x \quad y \quad z \\ \left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 3 & 0 & 0 & -2 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{array}\right] \end{array} \rightarrow \begin{array}{c} w \quad x \quad y \quad z \\ \left[\begin{array}{cccc|c} 1 & 0 & 0 & -2/3 & 0 \\ 0 & 1 & 0 & -1/2 & 0 \\ 0 & 0 & 1 & -1/3 & 0 \end{array}\right] \end{array}$$

$z = t$
$w = 2/3\, t$
$x = 1/2\, t$
$y = 1/3\, t$

Choose smallest non-negative integer solution

$t = 6$: $(w, x, y, z) = (4, 3, 2, 6)$

$$4NH_3 + 3O_2 \rightarrow 2N_2 + 6H_2O$$

Ex 4: a) **Inflow = Outflow at each node**

A: $20 = f_1 + f_2$

B: $f_1 + f_3 = 25$

C: $30 = f_3 + f_4$

D: $f_2 + f_4 = 25$

$$\begin{array}{cccc|c} f_1 & f_2 & f_3 & f_4 & \\ \hline 1 & 1 & 0 & 0 & 20 \\ 1 & 0 & 1 & 0 & 25 \\ 0 & 0 & 1 & 1 & 30 \\ 0 & 1 & 0 & 1 & 25 \end{array} \rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -5 \\ 0 & 1 & 0 & 1 & 25 \\ 0 & 0 & 1 & 1 & 30 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$f_4 = t$

$$\boxed{\begin{aligned} f_1 &= -5 + t \\ f_2 &= 25 - t \\ f_3 &= 30 - t \\ f_4 &= t \end{aligned}}$$

all $\geq 0 \Rightarrow$ $5 \leq t \leq 25$

b) $(f_1, f_2, f_3, f_4) = (5, 15, 20, 10)$

c) $5 \leq t \leq 25 \Rightarrow$ $0 \leq f_1 \leq 20$, $0 \leq f_2 \leq 20$, $5 \leq f_3 \leq 25$, $5 \leq f_4 \leq 25$

Ex 5: Let x = # of nickels
y = dimes
z = quarters

# of coins: $x + y + z = 20$

¢ : $5x + 10y + 25z = 300$

$$\left[\begin{array}{ccc|c} x & y & z & \\ 1 & 1 & 1 & 20 \\ 5 & 10 & 25 & 300 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -3 & -20 \\ 0 & 1 & 4 & 40 \end{array}\right]$$

$$\boxed{\begin{array}{c} z = t \\ x = -20 + 3t \\ y = 40 - 4t \end{array}} \leftarrow t \geq \frac{20}{3} \Rightarrow t \geq 7$$

$t \leq 10$

$$7 \leq t \leq 10$$