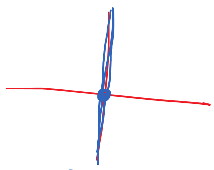
251DX02Mar26

March 26, 2021 1:26 PM

3.5 #1

Show that $\left\{ \begin{bmatrix} x \\ y \end{bmatrix} \text{ such that } x=0 \right\}$

is a subspace of $\mathbb{R}^2$.



$x=0$

Any line or plane through the origin is a subspace of $\mathbb{R}^n$.

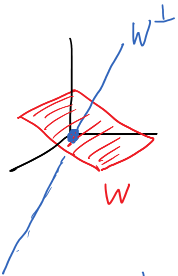
5.2 Orthogonal Complements and Projections

$W =$ subspace of $\mathbb{R}^n$

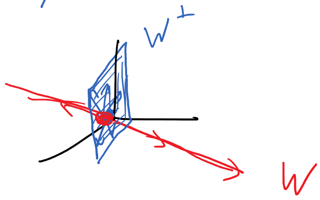
The <u>orthogonal complement</u> of W:

$$W^{\perp} = \{ \bar{v} \text{ in } \mathbb{R}^n \text{ such that } \bar{v} \cdot \bar{w} = 0 \text{ for all } \bar{w} \text{ in } W \}$$

<u>Ex</u>:



<u>Ex</u>:



<u>Ex</u>: W is a 2D plane through origin in $\mathbb{R}^4$

$W^{\perp}$ "

<u>Ex</u>: $W = \operatorname{span}\left(\begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 3 \end{bmatrix} \right)$

Find a basis for $W^{\perp}$.

Solve $\left[\begin{array}{cccc|c} 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & 3 & 0 \end{array}\right]$

$R_1 - 2R_2$ $\quad \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array}$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & -6 & 0 \\ 0 & 1 & 1 & 3 & 0 \end{array}\right] \quad \text{RREF}$$

$x_3 = s \qquad x_4 = t$

$x_1 - x_3 - 6x_4 = 0 \rightarrow x_1 = s + 6t$

$x_2 + x_3 + 3x_4 = 0 \rightarrow x_2 = -s - 3t$

$$\vec{x} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix} s + \begin{bmatrix} 6 \\ -3 \\ 0 \\ 1 \end{bmatrix} t$$

Basis for $W^\perp = \left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 6 \\ -3 \\ 0 \\ 1 \end{bmatrix} \right\}$

all orthogonal ✓

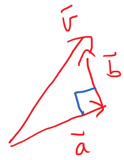
Check: Basis for $W = \left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 3 \end{bmatrix} \right\}$

Ex: W has orthogonal basis

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 1 \\ 3 \end{bmatrix} \right\}$$

Find the orthogonal decomposition of $\vec{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$ with respect to W.

$$\underset{(\text{in } W)}{\vec{v} = \vec{a}} + \underset{(\text{in } W^\perp)}{\vec{b}}$$



$\vec{a} = \text{proj}_W \vec{v}$

$= \text{proj}_{\vec{b}_1} \vec{v} + \text{proj}_{\vec{b}_2} \vec{v}$ (basis is orthogonal ✓)

$= \dfrac{\vec{b}_1 \cdot \vec{v}}{\|\vec{b}_1\|^2} \vec{b}_1 + \dots$

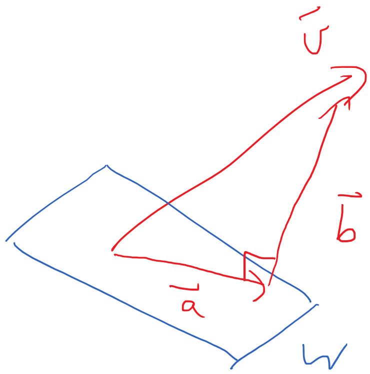
$= \dfrac{7}{5} \begin{bmatrix} 1 \\ 0 \\ 2 \\ 0 \end{bmatrix} + \cancel{\dfrac{15}{15}} \, \dfrac{5}{5} \begin{bmatrix} -2 \\ 1 \\ 1 \\ 3 \end{bmatrix}$

$= \dfrac{1}{5} \begin{bmatrix} -3 \\ 5 \\ 19 \\ 15 \end{bmatrix}$

$$\vec{b} = \vec{v} - \vec{a}$$

$$= \frac{5}{5}\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} - \frac{1}{5}\begin{bmatrix} -3 \\ 5 \\ 19 \\ 15 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} 8 \\ 5 \\ -4 \\ 5 \end{bmatrix}$$



<u>Ex</u>: $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}$

Find a basis for:

a) row(A)

RREF of $A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

$\{[1 \ 0 \ -1], [0 \ 1 \ 2]\}$

b) col(A)

RREF of $A = \begin{bmatrix} 1 & & \\ & 1 & \end{bmatrix}$

{columns 1 and 2 of A}

$\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} \right\}$

c) null(A)

Solve $A\vec{x} = \vec{0}$

$[A \mid \vec{0}]$

$$\left[\begin{array}{c|c} A & \vec{0} \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x_3 = t$

$x_1 = t$

$x_2 = -2t$

$$\vec{x} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} t$$

$$\left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \right\}$$

d) $[\text{row}(A)]^{\perp}$

$= \text{null}(A)$

$$\begin{bmatrix} \rule{2cm}{0.4pt} \\ \rule{2cm}{0.4pt} \\ \rule{2cm}{0.4pt} \end{bmatrix} \vec{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \right\}$$

e) $\text{null}(A^T)$

Solve $A^T \vec{x} = \vec{0}$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & 2 & 1 & 0 \\ 3 & 3 & 1 & 0 \end{array}\right]$$

$$\rightsquigarrow \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \text{ RREF}$$

$x_2 = t$

$x_1 = -t$

$x_3 = 0$

$$\bar{x} = \begin{bmatrix}-1\\1\\0\end{bmatrix} t$$

$$\left\{\begin{bmatrix}-1\\1\\0\end{bmatrix}\right\}$$

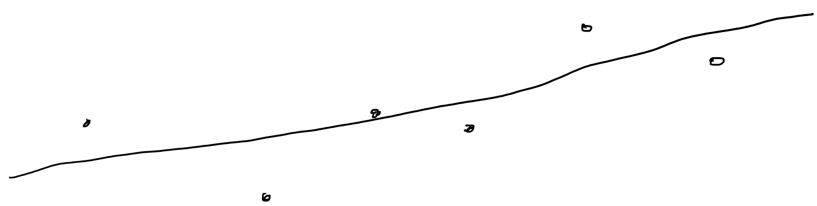
f) $[\text{Col}(A)]^\perp$

$$= [\text{row}(A^T)]^\perp$$

$$= \text{null}(A^T)$$

$$\left\{\begin{bmatrix}-1\\1\\0\end{bmatrix}\right\}$$

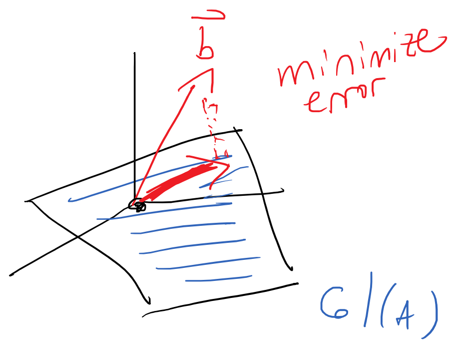
Why We Care (Section 7.3)



Find best-fit line.

Inconsistent system $A\bar{x} = \vec{b}$

$\Rightarrow$ $\vec{b}$ is not in Col(A)



Next 3 Weeks :

Ch5 Orthogonality

Complex Numbers

7.3 Best Fit Curves