251DX02Mar1

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Ex: Show that $\text{Span}\left(\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}2\\7\end{bmatrix}\right)$ is all of $\mathbb{R}^2$.

Let $c_1\begin{bmatrix}1\\2\end{bmatrix} + c_2\begin{bmatrix}2\\7\end{bmatrix} = \begin{bmatrix}a\\b\end{bmatrix}$

Show that system is consistent

$$\left[\begin{array}{cc|c}1 & 2 & a\\2 & 7 & b\end{array}\right]$$

$$R_2 - 2R_1 \quad \left[\begin{array}{cc|c}1 & 2 & a\\0 & 3 & b-2a\end{array}\right] \quad \text{REF}$$

System is consistent ✓

Follow-Up: Write $\begin{bmatrix}a\\b\end{bmatrix}$ as a linear combination of $\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}2\\7\end{bmatrix}$

$\vdots$

$$\left[\quad\right] \text{RREF}$$

3.1 #35

$$A = \begin{bmatrix}0 & 1\\-1 & 1\end{bmatrix}$$

a) Compute $A^2, \ldots, A^6$

b) What is A^{2001}?

$$A^2 = \begin{bmatrix}0 & 1\\-1 & 1\end{bmatrix}\begin{bmatrix}0 & 1\\-1 & 1\end{bmatrix} = \begin{bmatrix}-1 & 1\\-1 & 0\end{bmatrix}$$

$$A^3 = \begin{bmatrix}-1 & 1\\-1 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\-1 & 1\end{bmatrix} = \begin{bmatrix}-1 & 0\\0 & -1\end{bmatrix}$$

$\vdots$

$$A^6 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

b)

$$\begin{array}{r} 333 \\ 6 \overline{)\,2001} \\ -18 \\ \hline 20 \\ -18 \\ \hline 21 \\ 18 \\ \hline 3 \end{array} \leftarrow \text{remainder}$$

$$A^{2001} = A^{6(333)+3}$$
$$= A^{6(333)} A^3$$
$$= (A^6)^{333} A^3$$
$$= I^{333} A^3$$
$$= A^3$$

3.3 #23 Solve for X

$$ABXA^{-1}B^{-1} = I + A$$

$$\cancel{A^{-1}A}BXA^{-1}B^{-1} = A^{-1}(I+A)$$

$$\cancel{B^{-1}B}XA^{-1}B^{-1} = B^{-1}A^{-1}(I+A)$$

$$XA^{-1}\cancel{B^{-1}B} = B^{-1}A^{-1}(I+A)B$$

$$X\cancel{A^{-1}A} = \boxed{B^{-1}A^{-1}}(I+A)BA$$

$$X = (B^{-1}A^{-1} + B^{-1}A^{-1}A)\,BA$$

$$X = B^{-1}A^{-1}BA + B^{-1}\cancel{A^{-1}A}BA$$

$$X = B^{-1}A^{-1}BA + A$$

Ex: A bag of house blend contains 300g of Colombian and 100g of Kenyan

A bag of special blend contains 150g of Colombian and 250g of Kenyan.

a) Set up the System:

How many bags of each blend can be made with 3kg of Colombian and 2.2kg of Kenyan?

x = # bags of house blend

y = " special "

$$\begin{cases} \text{Colombian:} & 300x + 150y = 3000 \\ \text{Kenyan:} & 100x + 250y = 2200 \end{cases}$$

b) Solve using A^{-1}

$$A = \begin{bmatrix} 300 & 150 \\ 100 & 250 \end{bmatrix}$$

$$\det A = 60{,}000$$

$$A^{-1} = \frac{1}{60{,}000}\begin{bmatrix} 250 & -150 \\ -100 & 300 \end{bmatrix}$$

$$\vec{x} = A^{-1}\vec{b}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{60{,}000}\begin{bmatrix} 250 & -150 \\ -100 & 300 \end{bmatrix}\begin{bmatrix} 3000 \\ 2200 \end{bmatrix}$$

$$= \frac{1}{60{,}000}\begin{bmatrix} 420{,}000 \\ 360{,}000 \end{bmatrix}$$

$$= \begin{bmatrix} 7 \\ 6 \end{bmatrix}$$

Ex: Find k so that $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 4 \\ 7 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \\ k \end{bmatrix}$ are linearly dependent.

Let $c_1\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + c_2\begin{bmatrix} 4 \\ 4 \\ 7 \end{bmatrix} + c_3\begin{bmatrix} 5 \\ 2 \\ k \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 1 & 4 & 5 & 0 \\ 2 & 4 & 2 & 0 \\ 3 & 7 & k & 0 \end{array}\right]$$

$$\begin{array}{l} \\ R_2 - 2R_1 \\ R_3 - 3R_1 \end{array} \left[\begin{array}{ccc|c} 1 & 4 & 5 & 0 \\ 0 & -4 & -8 & 0 \\ 0 & -5 & k-15 & 0 \end{array}\right]$$

$$\frac{R_2}{-4} \left[\begin{array}{ccc|c} 1 & 4 & 5 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -5 & k-15 & 0 \end{array}\right]$$

$R_3 + 5R_2$ $\left[\begin{array}{ccc|c} 1 & 4 & 5 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & k-5 & 0 \end{array}\right]$ REF

∞-many solutions

$\Rightarrow$ $k=5$

Ex: Find the LU Factorization for $A = \begin{bmatrix} 4 & 2 & 8 \\ 1 & \frac{11}{2} & 12 \\ -8 & 0 & -5 \end{bmatrix}$

Current Row $- k$(Pivot Row)

$R_2 - \frac{1}{4}R_1$, $R_3 + 2R_1$ $\begin{bmatrix} 4 & 2 & 8 \\ 0 & 5 & 10 \\ 0 & 4 & 11 \end{bmatrix}$ $k = \frac{1}{4}$, $k = -2$

$R_3 - \frac{4}{5}R_2$ $\begin{bmatrix} 4 & 2 & 8 \\ 0 & 5 & 10 \\ 0 & 0 & 3 \end{bmatrix}$ $k = \frac{4}{5}$

$$\boxed{\begin{gathered} 4 - k(5) = 0 \\ k = \frac{4}{5} \end{gathered}}$$

$L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{4} & 1 & 0 \\ -2 & \frac{4}{5} & 1 \end{bmatrix}$ $U = \begin{bmatrix} 4 & 2 & 8 \\ 0 & 5 & 10 \\ 0 & 0 & 3 \end{bmatrix}$

Ex: Write A and A^{-1} as a product of elementary matrices.

$A = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$

$\frac{R_2}{-1}$ $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ $E_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ $E_1^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$R_1 - 2R_2$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $E_2 = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$ $E_2^{-1} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$(R_1 + 2R_2)$

$$\underbrace{E_2 E_1}_{A^{-1}} A = I$$

$$A^{-1} = E_2 E_1$$

$$A = (E_2 E_1)^{-1} = E_1^{-1} E_2^{-1}$$