

## 2.1 Linear Systems

Ex: Solve 
$$\begin{cases} 3x - 9y = 111 \\ 2x + y = -3 \end{cases}$$

$$\begin{array}{c} x \quad y \quad \# \\ \left[ \begin{array}{cc|c} 3 & -9 & 111 \\ 2 & 1 & -3 \end{array} \right] \end{array}$$

Get a 1

$$\frac{R_1}{3} \left[ \begin{array}{cc|c} 1 & -3 & 37 \\ 2 & 1 & -3 \end{array} \right]$$

Get 0's in rest of Column 1

$$R_2 - 2R_1 \left[ \begin{array}{cc|c} 1 & -3 & 37 \\ 0 & 7 & -77 \end{array} \right]$$

Current row  $-k$  (pivot row)

Get a 1

$$\frac{R_2}{7} \left[ \begin{array}{cc|c} 1 & -3 & 37 \\ 0 & 1 & -11 \end{array} \right]$$

Get 0's in rest of Column 2

$$R_1 + 3R_2 \left[ \begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & -11 \end{array} \right]$$

Current row  $-k$  (pivot row)

$$\begin{aligned} \cancel{x + 0y} &= 4 & x &= 4 \\ & & y &= -11 \end{aligned}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -11 \end{bmatrix}$$

Ex: Solve  $\begin{cases} 3x - 9y = 111 \\ 2x - 6y = 0 \end{cases}$

$$\begin{array}{cc|c} x & y & \# \\ \hline 3 & -9 & 111 \\ 2 & -6 & 0 \end{array}$$

Get a 1

$$\frac{R_1}{3} \begin{bmatrix} 1 & -3 & 37 \\ 2 & -6 & 0 \end{bmatrix}$$

Get 0's in rest of Column 1

$$R_2 - 2R_1 \begin{bmatrix} 1 & -3 & 37 \\ 0 & 0 & -74 \end{bmatrix}$$

Current row -k (pivot row)

$$0x + 0y = -74$$

IMPOSSIBLE

System has no solution.

Ex: Solve  $\begin{cases} -4x - 36y = -28 \\ x + 9y = 7 \end{cases}$

$$\begin{array}{cc|c} x & y & \# \\ \hline -4 & -36 & -28 \\ 1 & 9 & 7 \end{array}$$

Get a 1

$$R_1 \leftrightarrow R_2 \quad \left[ \begin{array}{cc|c} 1 & 9 & 7 \\ -4 & -36 & -28 \end{array} \right]$$

Get 0's in rest of Column 1

$$R_2 + 4R_1 \quad \left[ \begin{array}{cc|c} x & y & \# \\ 1 & 9 & 7 \\ 0 & 0 & 0 \end{array} \right]$$

Current row - k (pivot row)

Circle the leading nonzero entry in each row  
Any column without a circle is a free variable

$$\left[ \begin{array}{cc|c} 1 & 9 & 7 \\ 0 & 0 & 0 \end{array} \right]$$

$$y = k$$

$$x + 9y = 7 \rightarrow x + 9k = 7 \rightarrow x = 7 - 9k$$

$$\begin{cases} x = 7 - 9k \\ y = 0 + 1k \end{cases}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 0 \end{bmatrix} + k \begin{bmatrix} -9 \\ 1 \end{bmatrix}$$

Ex: Solve

$$\left[ \begin{array}{cc|c} x & y & \# \\ 1 & 2 & 10 \\ 3 & 6 & 30 \\ 12 & 24 & 120 \end{array} \right]$$

~~Get a 1~~

Get 0's in rest of Column 1

$$\begin{array}{l} R_2 - 3R_1 \\ R_3 - 12R_1 \end{array} \left[ \begin{array}{cc|c} x & y & \\ 1 & 2 & 10 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right]$$

current row  $-k$  (pivot row)

→ Circle the first nonzero entry in each row  
→ Columns without a circle are "free variables"

$$\left[ \begin{array}{cc|c} 1 & 2 & 10 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right]$$

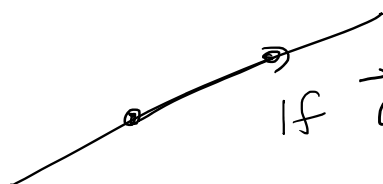
$$\boxed{y = k} \quad \text{parameter}$$

$$x + 2y = 10 \rightarrow x = 10 - 2y \rightarrow \boxed{x = 10 - 2k}$$

$$\begin{cases} x = 10 - 2k \\ y = 0 + 1k \end{cases}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \end{bmatrix} + k \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

In  $\mathbb{R}^2$



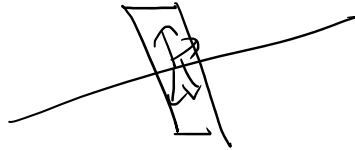
if  $\vec{d} = \begin{bmatrix} a \\ b \end{bmatrix}$

then  $\vec{n} = \begin{bmatrix} -b \\ a \end{bmatrix}$

$$\text{e.g. } \vec{d} = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \Rightarrow \vec{n} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

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In  $\mathbb{R}^3$



No normal vector  
for a line

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$$\begin{array}{cc} x & y \\ \left[ \begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{array} \right] \\ x=3 \\ y=4 \end{array}$$

$$\left[ \begin{array}{cc|c} 1 & 2 & 3 \\ 0 & 0 & 0 \end{array} \right]$$

$\infty$ -many  
Solutions