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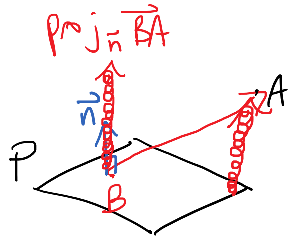
January 25, 2021 3:25 PM

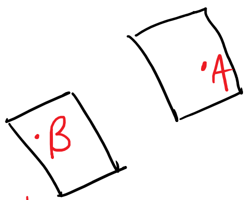
Ex: Find distance between point A and plane P



Choose point B on plane

distance $= \| \text{proj}_{\vec{n}} \vec{BA} \|$

Now consider parallel planes



Choose points A and B on either plane

1.4 CROSS PRODUCT (Part 2)

Determinants

$$\begin{vmatrix} 2 & 4 \\ 3 & 5 \end{vmatrix} = 2(5) - 4(3) = -2$$

sign

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ \cdots & \cdots & \cdots \end{vmatrix} = 1\begin{vmatrix} 5 & 6 \\ \cdots & \cdots \end{vmatrix} - 2\begin{vmatrix} 4 & 6 \\ \cdots & \cdots \end{vmatrix} + 3\begin{vmatrix} 4 & 5 \\ \cdots & \cdots \end{vmatrix}$$

$$\begin{vmatrix} 4 & 5 & 6 \\ 1 & 2 & -1 \end{vmatrix} = 1\begin{vmatrix} 5 & 6 \\ 2 & -1 \end{vmatrix} - 2\begin{vmatrix} 4 & 6 \\ 1 & -1 \end{vmatrix} + 3\begin{vmatrix} 4 & 5 \\ 1 & 2 \end{vmatrix}$$

$$= 1(-17) - 2(-10) + 3(3)$$

$$= 12$$

Ex: Calculate $[1,2,3] \times [4,5,6]$ two ways

a) $[-3, 6, -3]$

$$\begin{matrix} 1 & 2 & 3 & 1 & 2 \\ 4 & 5 & 6 & 4 & 5 \end{matrix}$$

b) $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix} = \vec{i}\begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} - \vec{j}\begin{vmatrix} 1 & 3 \\ 4 & 6 \end{vmatrix} + \vec{k}\begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix}$

sign

$$= \vec{i}(-3) - \vec{j}(-6) + \vec{k}(-3)$$

$$= -3[1,0,0] + 6[0,1,0] - 3[0,0,1]$$

$$= [-3, 6, -3]$$

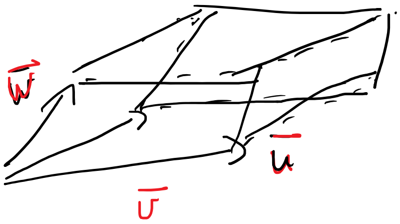
3 Geometry Formulas

1) $A(\text{parallelogram in } \mathbb{R}^3) = \| \vec{u} \times \vec{v} \|$

2) $A(\text{parallelogram in } \mathbb{R}^2) = \left| \det \begin{bmatrix} \vec{u} \\ \vec{v} \end{bmatrix} \right|$

3) $V(\text{parallelepiped in } \mathbb{R}^3) = \left| \det \begin{bmatrix} \vec{u} \\ \vec{v} \end{bmatrix} \right|$

3) $V(\text{parallelepiped in } \mathbb{R}^3) = \left| \det \begin{bmatrix} \vec{u} \\ \vec{v} \\ \vec{w} \end{bmatrix} \right|$



<u>Ex</u>: Area of the parallelogram determined by $\bar{u} = [7, x]$ and $\bar{v} = [3, 5]$?

$$\left| \det \begin{bmatrix} 7 & x \\ 3 & 5 \end{bmatrix} \right|$$

$$= |35 - 3x|$$

<u>Ex</u>: Area of the parallelogram determined by $\bar{u} = [1, 2, 3]$ and $\bar{v} = [4, -1, 6]$?

$$\|\bar{u} \times \bar{v}\|$$

$$= \|[15, 6, -9]\|$$

$$\boxed{\begin{array}{ccccc} 1 & 2 & 3 & 1 & 2 \\ 4 & -1 & 6 & 4 & -1 \end{array}}$$

$$= \sqrt{225 + 36 + 81}$$

$$= \sqrt{342} \quad \text{or} \quad 3\sqrt{38}$$

Ex: Volume of parallelepiped determined by

$$\bar{u} = [1, 2, 3]$$
$$\bar{v} = [9, 8, 7]$$
$$\bar{w} = [x, 3, 2] \quad ?$$

$$\left| \det \begin{bmatrix} \bar{u} \\ \bar{v} \\ \bar{w} \end{bmatrix} \right|$$

$$= \left| \det \begin{bmatrix} 1 & 2 & 3 \\ 9 & 8 & 7 \\ x & 3 & 2 \end{bmatrix} \right|$$

$$= \left| \; 1 \begin{vmatrix} 8 & 7 \\ 3 & 2 \end{vmatrix} - 2 \begin{vmatrix} 9 & 7 \\ x & 2 \end{vmatrix} + 3 \begin{vmatrix} 9 & 8 \\ x & 3 \end{vmatrix} \; \right|$$

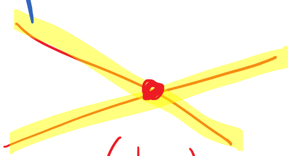
$$= \left| \; 1(-5) - 2(18 - 7x) + 3(27 - 8x) \right|$$

$$= \left| \; -5 - 36 + 14x + 81 - 24x \; \right|$$

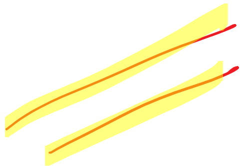
$$= \left| \; 40 - 10x \; \right|$$

Ch 2 Solving Systems

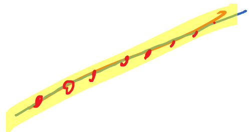
2 Equations in 2 Variables:



(x, y)



no solution



∞-many solutions

Then Bigger Systems:

$$\begin{cases} 2x - y + 4z = 9 \\ \cdots \\ \cdots \end{cases}$$

$$\begin{array}{cccc} x & y & z & \\ \hline \end{array}$$

$$\left[\begin{array}{ccc|c} 2 & -1 & 4 & 9 \\ \cdots & \cdots & \cdots & \\ \cdots & \cdots & \cdots & \end{array}\right]$$