251DX02Jan22

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1.4 Cross Product

Ex: $\vec{a} = [1, -2, 7]$ $\vec{b} = [6, 4, 3]$

$$\vec{a} \times \vec{b} = [-2(3) - 7(4), 7(6) - 1(3), 1(4) - (-2)(6)]$$
$$= [-34, 39, 16]$$

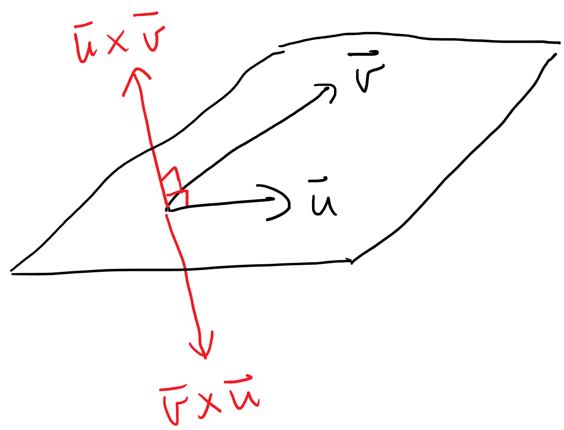
$$\begin{matrix} 1 & -2 & 7 & 1 & -2 \\ 6 & 4 & 3 & 6 & 4 \end{matrix}$$

Facts

$\vec{v} \times \vec{u} = -(\bar{u} \times \bar{v})$

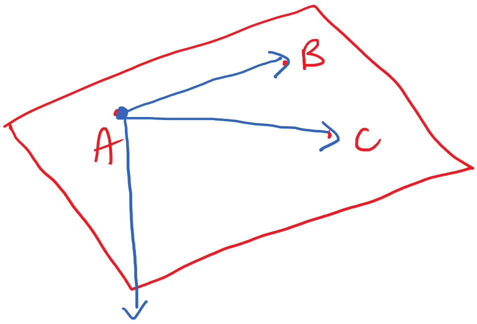
$\bar{u} \times \bar{v}$ is orthogonal to both $\bar{u}$ and $\bar{v}$

Right Hand Rule



Ex: General form of the plane

through $A = (2,1,3)$
$B = (4,4,4)$
$C = (5,6,7)$?



$\vec{n} = \vec{AB} \times \vec{AC}$

$$\vec{AB} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$
$$\vec{AC} = \begin{bmatrix} 3 \\ 5 \\ 4 \end{bmatrix}$$

$$\vec{n} = \vec{AB} \times \vec{AC} = [7, -5, 1]$$

$$\begin{array}{cccccc} 2 & 3 & 1 & 2 & 3 \\ 3 & 5 & 4 & 3 & 5 \end{array}$$

Normal

$$\vec{n} \cdot \vec{x} = \vec{n} \cdot \vec{p}$$

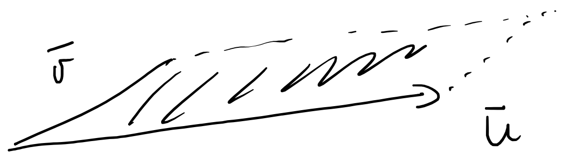
$$\begin{bmatrix} 7 \\ -5 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -5 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

General

$$7x - 5y + z = 12$$

FACT

$\boxed{\|\bar{u} \times \bar{v}\| = \text{area of the \underline{parallelogram} formed by } \bar{u} \text{ and } \bar{v}}$



FACT

$\frac{1}{2}\|\bar{u} \times \bar{v}\|$ = area of the triangle formed by $\bar{u}$ and $\bar{v}$

Ex: Area of the parallelogram determined by $\bar{u} = [1, -1, 2]$
$\bar{v} = [1, 1, 1]$?

$\bar{u} \times \bar{v} = [-3, 1, 2]$

$$\begin{array}{ccccc} 1 & -1 & 2 & 1 & -1 \\ 1 & 1 & 1 & 1 & 1 \end{array}$$

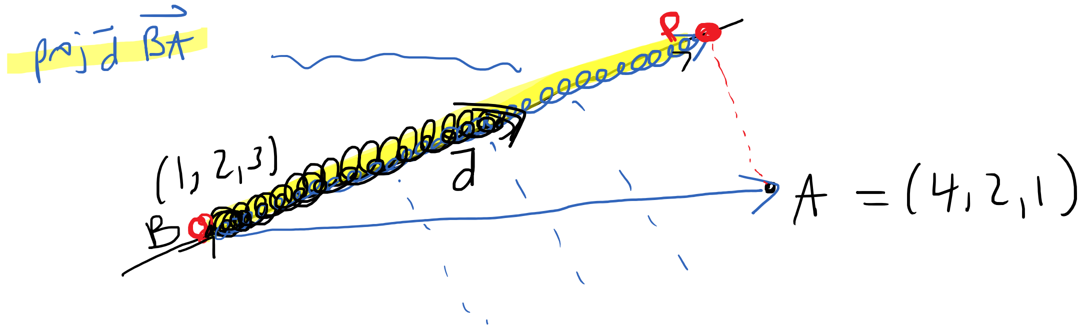
$\|\bar{u} \times \bar{v}\| = \sqrt{9 + 1 + 4}$
$= \sqrt{14}$

A(▱) $= \sqrt{14}$

Section 1.3

<u>Ex</u>: Consider $\vec{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + t\begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$

Find the point P on the line that is closest to $A=(4,2,1)$



$$\overrightarrow{BA} = \begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix}$$

$$\vec{d} = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$$

$$\text{proj}_{\vec{d}}\, \overrightarrow{BA} = \frac{\vec{d} \cdot \overrightarrow{BA}}{\|\vec{d}\|^2}\, \vec{d}$$

$$= -\frac{8}{14} \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$$

$$\vec{B} + \text{proj}_{\vec{d}}\, \overrightarrow{BA} = \vec{P}$$

$$\vec{P} = \vec{B} + \text{proj}_{\vec{d}}\, \overline{BA}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - \frac{8}{14} \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$$

$$= \frac{14}{14} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - \frac{8}{14} \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$$

$$= \frac{1}{14} \begin{bmatrix} 30 \\ 4 \\ 34 \end{bmatrix} \leftarrow 14(1) - 8(-2)$$

$$P = \left(\frac{30}{14}, \frac{4}{14}, \frac{34}{14} \right)$$