Section 1.3

Ex: Vector and parametric form of the line $2x+5y=10$?

Get 2 points on line

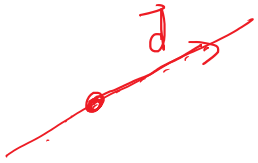
$P=(0,2)$

$Q=(5,0)$

$\vec{d} = \overrightarrow{PQ} = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$

VECTOR $\vec{x} = \vec{p} + t\vec{d}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} + t\begin{bmatrix} 5 \\ -2 \end{bmatrix}$

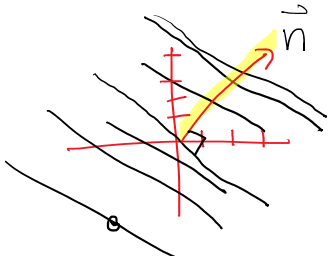


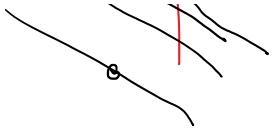
PARAMETRIC $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} + \begin{bmatrix} 5t \\ -2t \end{bmatrix}$

$$\begin{cases} x = 0+5t \\ y = 2-2t \end{cases}$$

Ex: A line in $\mathbb{R}^2$ has normal vector $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ and passes through $(-5,-7)$.
General and normal form?

normal vector: vector that's perpendicular to the object.





NORMAL FORM $\vec{n} \cdot \vec{x} = \vec{n} \cdot \vec{p}$

$$\begin{bmatrix} 3 \\ 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \cdot \begin{bmatrix} -5 \\ -7 \end{bmatrix}$$

$\vec{p}$ = vectorization of a point

GENERAL FORM $3x + 4y = -43$

Ex: Which of the 4 forms are useful for a line in $\mathbb{R}^3$?

Vector $\vec{x} = \vec{p} + t\vec{d}$

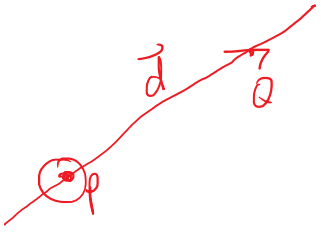
and

Parametric $\begin{cases} x = \\ y = \\ z = \end{cases}$

General and Normal form are not useful.

Ex: A line passes through $P=(1,2,3)$ and $Q=(-4,0,7)$
Vector and parametric form?

$$\vec{d} = \overrightarrow{PQ} = \begin{bmatrix} -5 \\ -2 \\ 4 \end{bmatrix}$$



VECTOR $\vec{x} = \vec{p} + t\vec{d}$

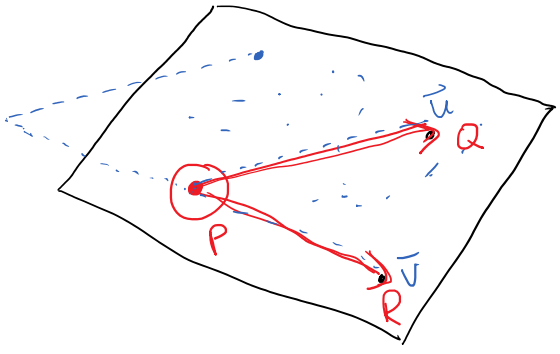
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + t \begin{bmatrix} -5 \\ -2 \\ 4 \end{bmatrix}$$

PARAMETRIC

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} -5t \\ -2t \\ 4t \end{bmatrix}$$

$$\begin{cases} x = 1 - 5t \\ y = 2 - 2t \\ z = 3 + 4t \end{cases}$$

Ex: Vector and parametric forms for plane through $P = (1, 0, 9)$, $Q = (-2, 4, 1)$, $R = (3, 6, 2)$



VECTOR $\vec{x} = \vec{p} + s\vec{u} + t\vec{v}$

direction vectors

$$\vec{u} = \overrightarrow{PQ} = \begin{bmatrix} -3 \\ 4 \\ -8 \end{bmatrix}$$

$$\vec{v} = \overrightarrow{PR} = \begin{bmatrix} 2 \\ 6 \\ -7 \end{bmatrix}$$

$$\vec{x} = \vec{p} + s\vec{u} + t\vec{v}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 9 \end{bmatrix} + s\begin{bmatrix} -3 \\ 4 \\ -8 \end{bmatrix} + t\begin{bmatrix} 2 \\ 6 \\ -7 \end{bmatrix}$$

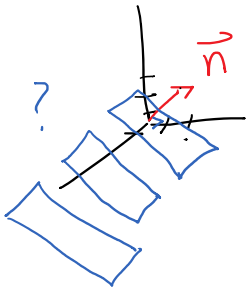
PARAMETRIC

$$\begin{cases} x = 1 - 3s + 2t \\ y = 0 + 4s + 6t \\ z = 9 - 8s - 7t \end{cases}$$

Ex: A plane in $\mathbb{R}^3$ has normal $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and passes through $(-1, -2, 6)$.
General and normal form?

normal = normal vector

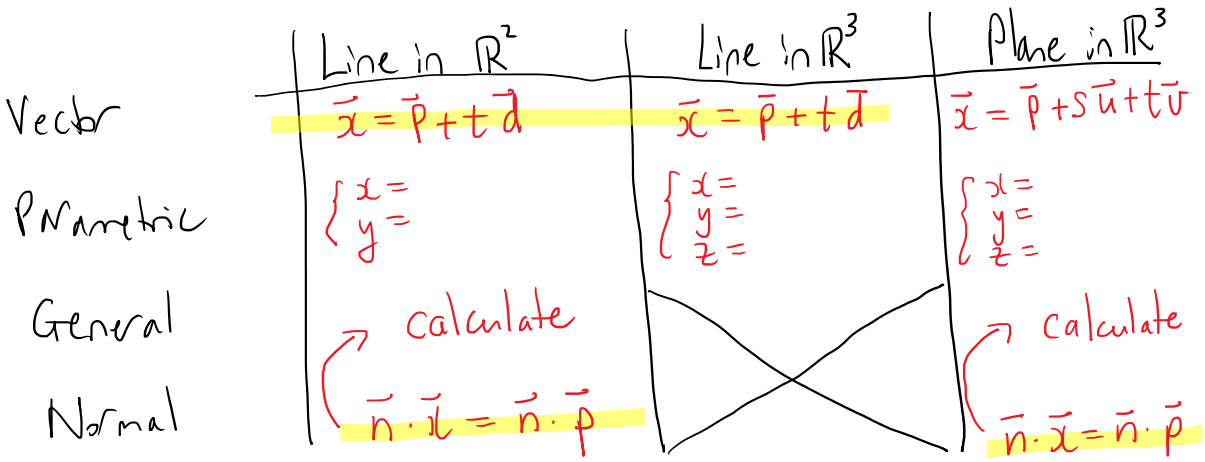
= vector that's perpendicular to the object



NORMAL $\quad \vec{n} \cdot \vec{x} = \vec{n} \cdot \vec{p}$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ -2 \\ 6 \end{bmatrix}$$

GENERAL $\quad x + 2y + 3z = 13$


Line in $\mathbb{R}^2$
Line in $\mathbb{R}^3$
Plane in $\mathbb{R}^3$
Vector
$\vec{x} = \vec{p} + t\vec{d}$
$\vec{x} = \vec{p} + t\vec{d}$
$\vec{x} = \vec{p} + s\vec{u} + t\vec{v}$
Parametric
$\begin{cases} x = \\ y = \end{cases}$
$\begin{cases} x = \\ y = \\ z = \end{cases}$
$\begin{cases} x = \\ y = \\ z = \end{cases}$
General
Normal
calculate
$\vec{n} \cdot \vec{x} = \vec{n} \cdot \vec{p}$
calculate
$\vec{n} \cdot \vec{x} = \vec{n} \cdot \vec{p}$