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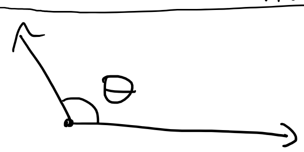
January 18, 2021 3:26 PM

Section 1.2

- Angles
- Projections

ANGLE



$0^\circ \le \theta \le 180^\circ$

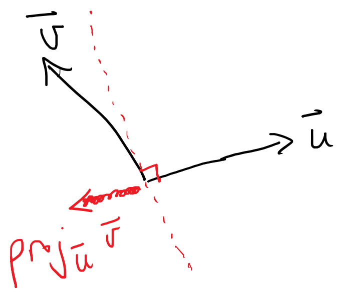
$$\vec{u}\cdot\vec{v} = \|\vec{u}\|\,\|\vec{v}\|\cos\theta$$

PROJECTIONS

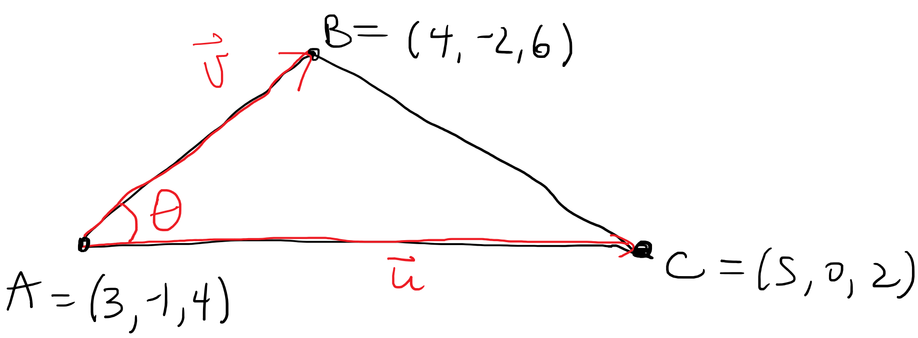
"Projection of $\vec{v}$ onto $\vec{u}$"

or "Projection onto $\vec{u}$ of $\vec{v}$"

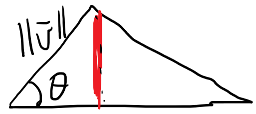
$$\text{proj}_{\vec{u}}\,\vec{v} = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|^2}\,\vec{u}$$



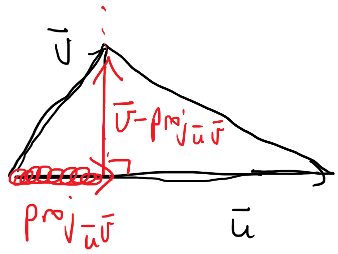
Section 1.2 #47



Calculate the area 2 ways:

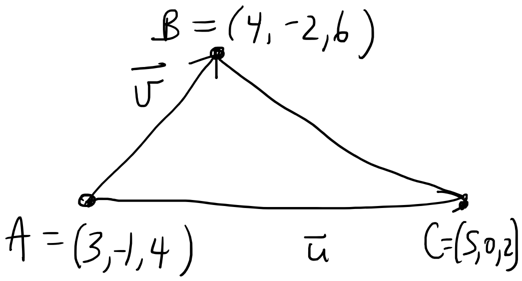


1) $A = \frac{1}{2}\|\bar{u}\| \underbrace{\|\bar{v}\|\sin\theta}_{h}$



2) $A = \frac{1}{2}\|\bar{u}\| \underbrace{\|\bar{v} - \text{proj}_{\bar{u}}\bar{v}\|}_{h}$

Calculation 1)



$$\bar{u} = \overrightarrow{AC} = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$$

$$\|\bar{u}\| = \sqrt{4 + 1 + 4} = \sqrt{9}$$

$$\bar{v} = \overrightarrow{AB} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

$$\|\bar{v}\| = \sqrt{1 + 1 + 4} = \sqrt{6}$$

$$\bar{u} \cdot \bar{v} = \|\bar{u}\|\|\bar{v}\|\cos\theta \qquad (*)$$

$$-3 = \sqrt{9}\sqrt{6}\cos\theta$$

$$\cos\theta = \frac{-\cancel{3}}{\cancel{\sqrt{9}}\sqrt{6}} = \frac{-1}{\sqrt{6}}$$

$$\sin\theta = \sqrt{1 - \cos^2\theta}$$

$$= \sqrt{1 - \left(\frac{-1}{\sqrt{6}}\right)^2}$$

$$= \sqrt{1 - \frac{1}{6}}$$

$$= \sqrt{\frac{5}{6}}$$

$$= \frac{\sqrt{5}}{\sqrt{6}}$$

$$A = \frac{1}{2} \|\bar{u}\| \|\bar{v}\| \sin\theta$$

$$= \frac{1}{2}\sqrt{9}\,\cancel{\sqrt{6}}\,\frac{\sqrt{5}}{\cancel{\sqrt{6}}}$$

$$= \frac{\sqrt{45}}{2}$$

Calculation 2)

$$A = \frac{1}{2}\|\bar{u}\| \|\bar{v} - \text{proj}_{\bar{u}}\bar{v}\|$$

Recall $\bar{u} = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$

$\bar{v} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$

$$\text{proj}_{\bar{u}}\bar{v} = \frac{\bar{u}\cdot\bar{v}}{\|\bar{u}\|^2}\bar{u} \qquad \textcircled{*}$$

$$= \frac{-3}{9}\vec{u}$$

$$= -\frac{1}{3}\begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$$

$$\bar{v} - \text{proj}_{\bar{u}}\bar{v} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} + \frac{1}{3}\begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} 5/3 \end{bmatrix}$$

$$= \begin{bmatrix} 5/3 \\ -2/3 \\ 4/3 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix}$$

$$\|\vec{v} - \mathrm{proj}_{\vec{u}}\, \vec{v}\| = \frac{1}{3}\sqrt{45}$$

$$\boxed{\text{Handy Trick} \qquad \|c\vec{w}\| = |c|\,\|\vec{w}\|}$$

$$A = \frac{1}{2}\|\vec{u}\|\,\|\vec{v} - \mathrm{proj}_{\vec{u}}\vec{v}\|$$

$$= \frac{1}{2}\cancel{\sqrt{9}} \cdot \frac{1}{\cancel{3}}\sqrt{45}$$

$$= \frac{\sqrt{45}}{2}$$

1.2 #11

$$\vec{u} = [1, \sqrt{2}, \sqrt{3}, 0]$$

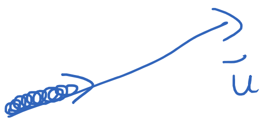
a) Find $\|\vec{u}\|$

b) Find a unit vector in direction of $\vec{u}$

a) $\|\vec{u}\| = \sqrt{1 + 2 + 3 + 0} = \sqrt{6}$

b) $\dfrac{\vec{u}}{\sqrt{6}}$

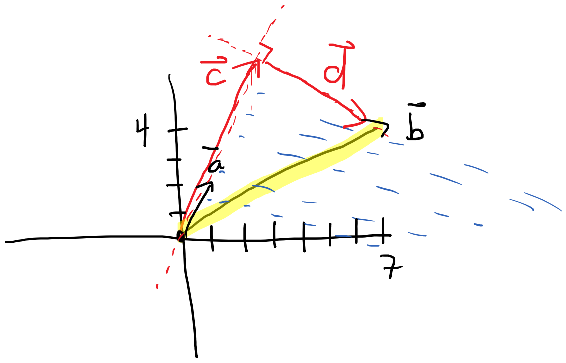
$$= \frac{1}{\sqrt{6}}[1, \sqrt{2}, \sqrt{3}, 0]$$



Ex: $\vec{a} = [1, 2]$ $\qquad \vec{b} = [7, 4]$

Find vectors $\vec{c}$ and $\vec{d}$ so that:

$$\begin{cases} \vec{b} = \vec{c} + \vec{d} \\ \vec{c} \text{ is parallel to } \vec{a} \\ \underline{\text{and}} \quad \vec{d} \text{ is perpendicular to } \vec{a} \end{cases}$$



$$\vec{c} = \operatorname{proj}_{\vec{a}} \vec{b}$$

$$= \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\|^2} \vec{a}$$

$$= \frac{15}{5} [1, 2]$$

$$= 3 [1, 2]$$

$$= [3, 6]$$

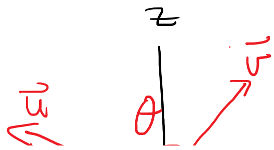
$$\vec{c} + \vec{d} = \vec{b}$$

$$\vec{d} = \vec{b} - \vec{c}$$

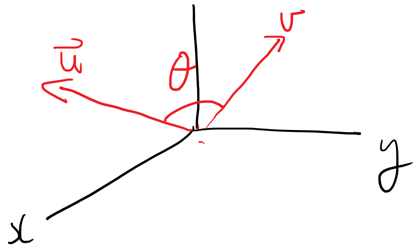
$$= [7, 4] - [3, 6]$$

$$= [4, -2]$$

<u>Ex:</u> Find the angle between $\vec{u} = \begin{bmatrix} 1 \\ -9 \\ 2 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 3 \\ 4 \\ 7 \end{bmatrix}$



$\theta \approx 110°$?



$\theta \approx 110°$?

$$\bar{u} \cdot \bar{v} = \|\bar{u}\| \, \|\bar{v}\| \cos\theta$$

$$-19 = \sqrt{86}\,\sqrt{74}\,\cos\theta$$

$$\frac{-19}{\sqrt{86}\,\sqrt{74}} = \cos\theta$$

$$\theta = \cos^{-1}\left(\frac{-19}{(\sqrt{86}\,\sqrt{74})}\right)$$

$$\approx 104°$$

Try to get comfortable with horizontal and vertical notation for vectors:

$$[1, 2, 3] + [4, 5, 6] = [5, 7, 9]$$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \\ 9 \end{bmatrix}$$