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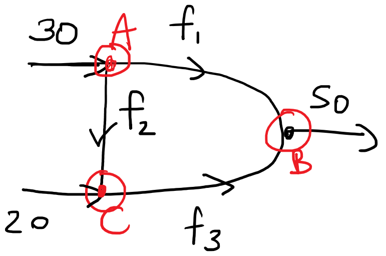
February 8, 2021 3:17 PM

Section 2.4

Ex:



e.g. vehicles/hour

Assume flows are non-negative
Find all possible flows.

Inflow = outflow

$$\begin{cases} A: & 30 = f_1 + f_2 \\ B: & f_1 + f_3 = 50 \\ C: & 20 + f_2 = f_3 \end{cases} \rightarrow \quad f_2 - f_3 = -20$$

$$\begin{array}{c} \begin{array}{ccc} f_1 & f_2 & f_3 \end{array} \\ \left[\begin{array}{ccc|c} 1 & 1 & 0 & 30 \\ 1 & 0 & 1 & 50 \\ 0 & 1 & -1 & -20 \end{array}\right] \end{array}$$

$$R_2 - R_1 \quad \left[\begin{array}{ccc|c} 1 & 1 & 0 & 30 \\ 0 & -1 & 1 & 20 \\ 0 & 1 & -1 & -20 \end{array}\right]$$

$$\frac{R_2}{(-1)} \quad \left[\begin{array}{ccc|c} 1 & 1 & 0 & 30 \\ 0 & 1 & -1 & -20 \\ 0 & 1 & -1 & -20 \end{array}\right]$$

$$\begin{array}{c} R_1 - R_2 \\ \\ R_3 - R_2 \end{array} \quad \begin{array}{c} \begin{array}{ccc} f_1 & f_2 & f_3 \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 1 & 50 \\ 0 & 1 & -1 & -20 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array} \quad \text{RREF}$$

$$\boxed{f_3 = t}$$

$$\boxed{f_3 = t}$$

$$f_1 + f_3 = 50 \rightarrow \boxed{f_1 = 50 - t}$$

$$f_2 - f_3 = -20 \rightarrow \boxed{f_2 = -20 + t}$$

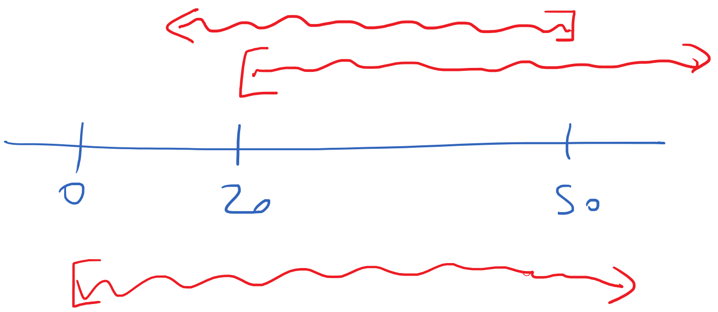
Ensure flows are non-negative:

f_1 : $50 - t \geq 0 \rightarrow 50 \geq t \rightarrow t \leq 50$

AND f_2 : $-20 + t \geq 0 \rightarrow t \geq 20$

AND f_3 : $t \geq 0$

$$\boxed{20 \leq t \leq 50}$$



$$\begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} = \begin{bmatrix} 50 \\ -20 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \qquad (20 \leq t \leq 50)$$

Ex: Set up, but don't solve

UNITS OF FOOD CONSUMED PER DAY

	Strain I	II	III
Food A	2	2	1
B	1	2	0
C	1	3	1

Lab has 2300 units of Food A
800 " B
1500 " C

How many bacteria of each strain can consume all of the food in one day?

Let x = # of bacteria of Strain I
y = " II
z = " III

Equations:

Food A: $2x + 2y + 4z = 2300$

B: $x + 2y = 800$

C: $x + 3y + z = 1500$

FYI $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 100 \\ 350 \\ 350 \end{bmatrix}$

Section 2.3 Linear Independence

Consider $c_1 \bar{u} + c_2 \bar{v} + c_3 \bar{w} = \bar{0}$

$c_1 = c_2 = c_3 = 0$

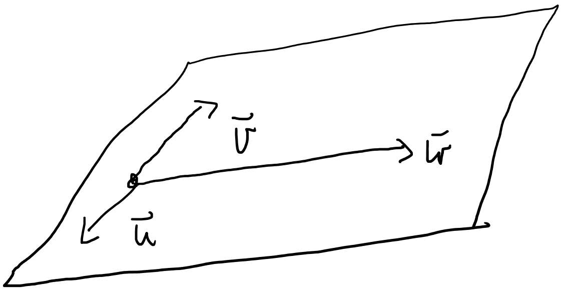
$\boxed{\bar{u}, \bar{v}, \bar{w} \text{ are linearly } \underline{\text{independent}}}$

$c_1 = c_2 = c_3 = 0$ and other solutions

$\boxed{\bar{u}, \bar{v}, \bar{w} \text{ are linearly } \underline{\text{dependent}}}$

Quick Ex:

a)



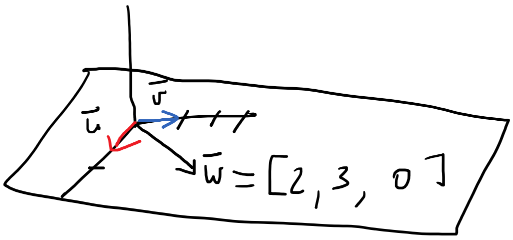
$\bar{v} = -2\bar{u}$

$2\bar{u} + \bar{v} + 0\bar{w} = \bar{0}$

$\bar{u}, \bar{v}, \bar{w}$ are linearly <u>dependent</u>

$\{\bar{u}, \bar{v}, \bar{w}\}$ is linearly <u>dependent</u>

b)



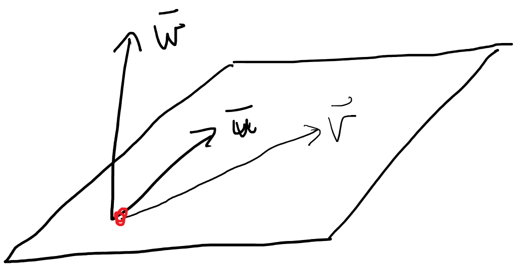
$\bar{w} = 2\bar{u} + 3\bar{v}$

$-2\bar{u} - 3\bar{v} + \bar{w} = \bar{0}$

$\{\bar{u}, \bar{v}, \bar{w}\}$ is linearly <u>dependent</u>

$\bar{u}, \bar{v}, \bar{w}$ are "

c)



$\{\bar{u}, \bar{v}, \bar{w}\}$ is linearly <u>independent</u>

$\bar{u}, \vec{v}, \bar{w}$ are "

$$C_1\bar{u} + C_2\bar{v} + C_3\bar{w} = \bar{0}$$

$\Rightarrow c_1 = c_2 = c_3 = 0$

<u>Ex</u>: a) $\{\bar{u}, \bar{v}, \bar{w}\}$ is linearly independent

Is $\{\bar{u}, \bar{v}\}$ linearly independent?

$\boxed{\text{YES}}$

GIVEN: $c_1\bar{u} + c_2\bar{v} + c_3\bar{w} = \bar{0} \Rightarrow c_1 = 0, c_2 = 0, c_3 = 0$

CONCLUDE: $c_1\bar{u} + c_2\bar{v} = \bar{0} \Rightarrow c_1 = 0, c_2 = 0$

b) $\{\bar{u}, \bar{v}, \bar{w}\}$ is linearly independent.

Is $\{\bar{u}, \bar{v}, \bar{w}, \bar{x}\}$ linearly independent?

$\boxed{\text{MAYBE}}$

e.g. $\bar{x} = 2\bar{u} \to -2\bar{u} + 0\bar{v} + 0\bar{w} + \bar{x} = \bar{0}$

Linearly Dependent

e.g. $\bar{u} = [1, 0, 0, 0]$ $\bar{v} = [0, 1, 0, 0]$ $\bar{w} = [0, 0, 1, 0]$

$\bar{x} = [0, 0, 0, 1]$ Linearly Independent

$c_1\bar{u} + c_2\bar{v} + c_3\bar{w} + c_4\bar{x} = \bar{0}$

$\Rightarrow c_1 = c_2 = c_3 = c_4 = 0$