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Do Suggested Homework!

2.3 Span and Linear Independence

(2nd hardest section in the course)

Linear Combination of $\bar{u}$ and $\bar{v}$:

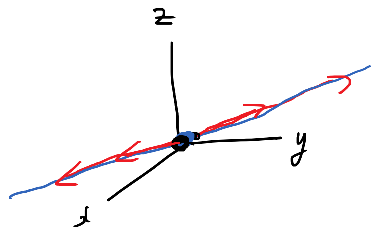
$$c_1\vec{u} + c_2\vec{v}$$

Span of $\bar{u}$ and $\bar{v}$: The set of all linear combinations of $\bar{u}$ and $\bar{v}$

$$\{c_1\bar{u} + c_2\bar{v}\}$$

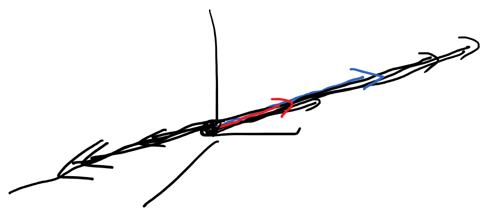
Quick Ex:

a) $\text{span}\left(\begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}\right)$ = line in $\mathbb{R}^3$ through origin



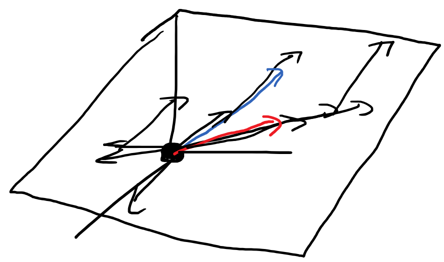
b) $\text{span}\left(\begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 6 \\ 2 \end{bmatrix}\right)$

= line in $\mathbb{R}^3$ through origin



c) $\text{span}\left(\begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix} \right)$

$=$ plane in $\mathbb{R}^3$ through origin



Ex: Vector and normal form of

$$\underbrace{\text{span}\left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \right)}_{\text{plane in } \mathbb{R}^3 \text{ through origin}} \ ?$$

Every span contains the origin:

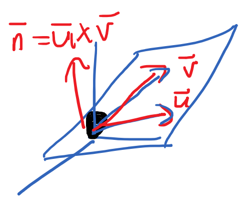
$$0\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 0\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Vector form $\qquad \vec{x} = \bar{p} + s\bar{u} + t\bar{v}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + t\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

Normal form:

Find $\bar{n}$:



$\bar{n} = \bar{u} \times \bar{v}$

$= [-3, 6, -3]$

1 2 3 1 2
4 5 6 4 5

$\bar{n} \cdot \bar{x} = \bar{n} \cdot \bar{p}$

$$\begin{bmatrix} -3 \\ 6 \\ -3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ 6 \\ -3 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Ex: Show that $\operatorname{span}\left(\begin{bmatrix} 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 7 \\ 8 \end{bmatrix}\right) = \mathbb{R}^2$

Let $\begin{bmatrix} a \\ b \end{bmatrix}$ be any vector in $\mathbb{R}^2$.

Show that $c_1 \begin{bmatrix} 5 \\ 6 \end{bmatrix} + c_2 \begin{bmatrix} 7 \\ 8 \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$ is solvable.

$c_1(5) + c_2(7) = a$

$$\begin{array}{c} \begin{array}{cc} c_1 & c_2 \end{array} \\ \left[\begin{array}{cc|c} 5 & 7 & a \\ 6 & 8 & b \end{array}\right] \end{array}$$

$\frac{R_1}{5}$
$$\left[\begin{array}{cc|c} 1 & \frac{7}{5} & \frac{a}{5} \\ 6 & 8 & b \end{array}\right]$$

$R_2 - 6R_1$
$$\begin{array}{c} \begin{array}{cc} c_1 & c_2 \end{array} \\ \left[\begin{array}{cc|c} 1 & \frac{7}{5} & \frac{a}{5} \\ 0 & -\frac{2}{5} & b - \frac{6a}{5} \end{array}\right] \end{array}$$ REF

$\boxed{8 - \frac{42}{5}}$

System is solvable.

$$\boxed{\begin{array}{l} \text{No solution} \\ \left[\begin{array}{cc|c} 0 & 0 & \text{nonzero} \end{array}\right] \end{array}}$$

Linear Independence

Consider $c_1 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 5 \\ 7 \end{bmatrix} + c_3 \begin{bmatrix} -2 \\ 1 \\ 6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Only solution is
$c_1 = c_2 = c_3 = 0$

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$c_1 = c_2 = c_3 = 0$
and other solutions

$\boxed{\text{Vectors are (linearly) \underline{independent}}}$

$\boxed{\text{Vectors are linearly \underline{dependent}}}$

<u>Ex</u>: Are $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 5 \\ 7 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ 1 \\ 6 \end{bmatrix}$ linearly independent?

$$c_1 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 5 \\ 7 \end{bmatrix} + c_3 \begin{bmatrix} -2 \\ 1 \\ 6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{array}{c} \begin{array}{ccc} c_1 & c_2 & c_3 \end{array} \\ \left[\begin{array}{ccc|c} 1 & 2 & -2 & 0 \\ 2 & 5 & 1 & 0 \\ 3 & 7 & 6 & 0 \end{array}\right] \end{array}$$

$$\begin{array}{r} \\ R_2 - 2R_1 \\ R_3 - 3R_1 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & -2 & 0 \\ 0 & 1 & 5 & 0 \\ 0 & 1 & 12 & 0 \end{array}\right]$$

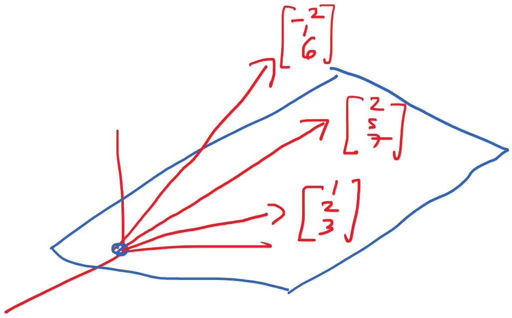
$$\begin{array}{r} R_1 - 2R_2 \\ \\ R_3 - R_2 \end{array} \begin{array}{c} \begin{array}{ccc} c_1 & c_2 & c_3 \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & -12 & 0 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & 7 & 0 \end{array}\right] \end{array}$$

REF

$$7c_3 = 0 \rightarrow \boxed{c_3 = 0}$$

$$c_2 + 5c_3 = 0 \rightarrow \boxed{c_2 = 0}$$

$$c_1 - 12c_3 = 0 \rightarrow \boxed{c_1 = 0}$$



$\boxed{\text{YES}}$