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7.3 Best-Fit Curves

Ex: Find the best-fit line through

x	y
0	4
2	7
3	9

$$y = a_0 + a_1 x$$

(a_0, a_1 are the variables)

$$1(a_0) + x(a_1) = y$$

$$\begin{array}{cc} 1 & x \end{array} \qquad\qquad y$$
$$\begin{bmatrix} 1 & 0 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \\ 9 \end{bmatrix}$$

$$\vec{x}^* = (A^T A)^{-1} A^T \vec{b}$$

$$A^T A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 5 & 13 \end{bmatrix}$$

$$(A^T A)^{-1} = \frac{1}{14} \begin{bmatrix} 13 & -5 \\ -5 & 3 \end{bmatrix}$$

$$\vec{x}^* = (A^T A)^{-1} A^T \vec{b}$$

$$= \frac{1}{14} \begin{bmatrix} 13 & -5 \\ -5 & 3 \end{bmatrix} \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 7 \\ 9 \end{bmatrix}}$$

$$= \frac{1}{14} \begin{bmatrix} 13 & -5 \\ -5 & 3 \end{bmatrix} \begin{bmatrix} 20 \\ 41 \end{bmatrix}$$

$$= \frac{1}{14} \begin{bmatrix} 55 \\ 23 \end{bmatrix}$$

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 55/14 \\ 23/14 \end{bmatrix}$$

$$y = a_0 + a_1 x$$

$$y = \frac{55}{14} + \frac{23}{14} x$$

<u>Ex</u>: Find the best-fit curve $P = Ce^{kt}$ through:

t	P
1	7
2	27
4	360

$$P = Ce^{kt}$$

$$\ln P = \ln C + \cancel{\ln e^{kt}}\ kt$$

($\ln c$, k are variables)

$$1(\ln C) + tk = \ln P$$

$$\begin{array}{cc} 1 & t \end{array}$$

$$\begin{bmatrix} & \\ & \end{bmatrix} \begin{bmatrix} \ln c \\ k \end{bmatrix} = \begin{bmatrix} \ln P \\ \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 4 \end{bmatrix}}_{A} \begin{bmatrix} \ln c \\ k \end{bmatrix} = \underbrace{\begin{bmatrix} \ln 7 \\ \ln 27 \\ \ln 360 \end{bmatrix}}_{\vec{b}}$$

$$\vec{x}^* = (A^T A)^{-1} A^T \vec{b}$$

$$\vdots$$

$$\approx \begin{bmatrix} 0.651 \\ 1.311 \end{bmatrix}$$

$$\approx \begin{bmatrix} 0.651 \\ 1.311 \end{bmatrix}$$

$$\begin{bmatrix} \ln c \\ k \end{bmatrix} \approx \begin{bmatrix} 0.651 \\ 1.311 \end{bmatrix}$$

$$\boxed{\begin{aligned} &\ln c \approx 0.651 \\ &c \approx e^{0.651} \approx 2 \end{aligned}} \qquad \boxed{k \approx 1}$$

$$P = Ce^{kt}$$
$$P = 2e^{t}$$

Exam Review

Ex:

(Section 3.5)

A has RREF = $\begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$

Find rank(A) and nullity(A)

$\boxed{\text{\# nonzero rows in REF/RREF}}$

\# parameters in solution to $A\vec{x} = \vec{0}$

rank(A) = 3

nullity(A) = 2

nullity(A) = #columns − rank(A)

Ex:

(Section 2.4)

Balance

$$\underset{(w)}{FeS} + \underset{(x)}{O_2} \rightarrow \underset{(y)}{Fe_2O_3} + \underset{(z)}{SO_2}$$

3 equations:

Fe: $w = 2y \rightarrow w - 2y = 0$

S: $w = z \rightarrow w - z = 0$

$$\begin{cases} S: & w = z \to w - z = 0 \\ D: & 2x = 3y + 2z \to 2x - 3y - 2z = 0 \end{cases}$$

$$\begin{array}{cccc} w & x & y & z \end{array}$$
$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 0 & 2 & -3 & -2 & 0 \end{array}\right]$$

$$R_2 - R_1 \quad \left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & 0 & 2 & -1 & 0 \\ 0 & 2 & -3 & -2 & 0 \end{array}\right]$$

$$R_2 \leftrightarrow R_3 \text{ and } \frac{R_2}{2} \quad \left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & -\frac{3}{2} & -1 & 0 \\ 0 & 0 & 2 & -1 & 0 \end{array}\right]$$

$$\frac{R_3}{2} \quad \left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & -\frac{3}{2} & -1 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 0 \end{array}\right]$$

$$\begin{array}{l} R_1 + 2R_3 \\ R_2 + \frac{3}{2}R_3 \end{array} \quad \begin{array}{c} \begin{array}{cccc} w & x & y & z \end{array} \\ \left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -7/4 & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & 0 \end{array}\right] \end{array} \quad \text{RREF}$$

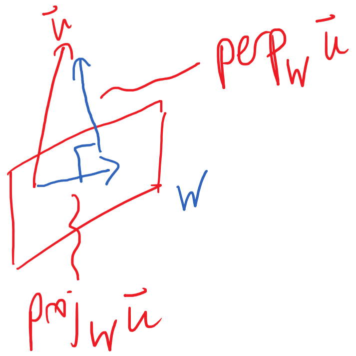
$$\left.\begin{array}{l} z = t \\ w = t \\ x = \frac{7}{4}t \\ y = \frac{1}{2}t \end{array}\right\} \text{non-negative integers}$$

Choose $t = 4$: $\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \\ 2 \\ 4 \end{bmatrix}$

$4FeS + 7O_2 \rightarrow 2Fe_2O_3 + 4SO_2$

Ex: $W = \text{span}\left(\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} -5 \\ 4 \\ 0 \end{bmatrix} \right)$

(Section 5.2) Find the orthogonal decomposition of $\bar{u} = \begin{bmatrix} 7 \\ 1 \\ 2 \end{bmatrix}$ with respect to W.



Notice that the basis for W is <u>orthogonal</u>

$$\text{proj}_W \vec{u} = \text{proj}_{\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}} \bar{u} + \text{proj}_{\begin{bmatrix} -5 \\ 4 \\ 0 \end{bmatrix}} \bar{u}$$

$$= \frac{45}{77} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \; \frac{-31}{41} \begin{bmatrix} -5 \\ 4 \\ 0 \end{bmatrix}$$

$\times \frac{41}{41}$ $\quad \times \frac{77}{77}$

$$= \frac{1845}{3157} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} - \frac{2387}{3157} \begin{bmatrix} -5 \\ 4 \\ 0 \end{bmatrix}$$

$$= \frac{1}{3157} \begin{bmatrix} 19315 \\ -323 \\ 11070 \end{bmatrix}$$

$$\text{proj}_W \bar{u} + \text{perp}_W \bar{u} = \bar{u}$$

$$\text{perp}_W \bar{u} = \bar{u} - \text{proj}_W \bar{u}$$

$$= \begin{bmatrix} 7 \\ 1 \\ 2 \end{bmatrix} - \frac{1}{3157} \begin{bmatrix} 19315 \\ -323 \\ 11070 \end{bmatrix}$$

$\times \frac{3157}{3157}$

$$= \frac{1}{3157}\begin{bmatrix} 2784 \\ 3480 \\ -4756 \end{bmatrix}$$

Optional Check:

$$(\text{proj}_{W}\,\bar{u}) \cdot (\text{perp}_{W}\,\bar{u}) = 0 \quad \checkmark$$