

Test: 1.1 - 1.4, 2.1 - 2.2 (1.4 = Cross Product)

5 Questions

45 minutes to solve  
15 " for scanning

SUBMIT TO D2L DROPBOX

Allowed

- Course website (notes/videos)
- Your personal notes

Not Allowed

- External websites (Google / Chegg)
- Communicate with others

## 2.2 Solving Systems

Ex: Solve by Gaussian Elimination

$$\begin{array}{ccc|c} x & y & z & \\ \hline 3 & 4 & 5 & 41 \\ 0 & 2 & 1 & 16 \\ 0 & 8 & 7 & 76 \end{array}$$

Matrix  $\rightarrow$  REF

Solve by backsubstitution

- REF:
- 1) Any zero rows @ bottom
  - 2) Leading nonzero entries in each row  
 $\searrow$  more down and right

$$\begin{array}{ccc|c} x & y & z & \\ \hline \dots & \dots & \dots & \dots \end{array}$$

$$R_3 - 4R_2 \quad \begin{array}{ccc|c} x & y & z & \\ \hline 3 & 4 & 5 & 41 \\ 0 & 2 & 1 & 16 \\ 0 & 0 & 3 & 12 \end{array} \text{ REF}$$

$$3z = 12 \rightarrow \boxed{z = 4}$$

$$2y + z = 16 \rightarrow 2y + 4 = 16 \rightarrow \boxed{y = 6}$$

$$3x + 4y + 5z = 41 \rightarrow 3x + 24 + 20 = 41 \rightarrow \boxed{x = -1}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 6 \\ 4 \end{bmatrix}$$

Ex: Solve by Gauss-Jordan Elimination

$$\begin{array}{ccc|c} x & y & z & \\ \hline 4 & 2 & 2 & 8 \\ 3 & 5 & 6 & 11 \\ 2 & 2 & 3 & 4 \end{array}$$

Matrix  $\rightarrow$  RREF

- RREF:
- 1) REF
  - 2) Pivots = 1
  - 3) Pivots have 0's everywhere else in their columns.

$$\frac{R_1}{4} \quad \begin{array}{ccc|c} 1 & \frac{1}{2} & \frac{1}{2} & 2 \\ \hline 3 & 5 & 6 & 11 \\ 2 & 2 & 3 & 4 \end{array}$$

$$\begin{array}{l} R_2 - 3R_1 \\ R_3 - 2R_1 \end{array} \quad \begin{array}{ccc|c} 1 & \frac{1}{2} & \frac{1}{2} & 2 \\ 0 & \frac{7}{2} & \frac{9}{2} & 5 \\ 0 & 1 & 2 & 0 \end{array} \quad \frac{10}{2} - \frac{3}{2}$$

Current row - k(pivot row)

$$R_2 \leftrightarrow R_3 \quad \left[ \begin{array}{ccc|c} 1 & \frac{1}{2} & \frac{1}{2} & 2 \\ 0 & \textcircled{1} & 2 & 0 \\ 0 & \frac{7}{2} & \frac{9}{2} & 5 \end{array} \right]$$

$$R_1 - \frac{1}{2}R_2 \quad \left[ \begin{array}{ccc|c} 1 & 0 & -\frac{1}{2} & 2 \\ 0 & \textcircled{1} & 2 & 0 \\ 0 & 0 & -\frac{5}{2} & 5 \end{array} \right]$$

$$R_3 - \frac{7}{2}R_2 \quad \left[ \begin{array}{ccc|c} 1 & 0 & -\frac{1}{2} & 2 \\ 0 & \textcircled{1} & 2 & 0 \\ 0 & 0 & -\frac{5}{2} & 5 \end{array} \right] \quad \frac{9}{2} - \frac{14}{2}$$

$$R_3 \times \left(\frac{-2}{5}\right) \quad \left[ \begin{array}{ccc|c} 1 & 0 & -\frac{1}{2} & 2 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & \textcircled{1} & -2 \end{array} \right] \quad 5 \left(\frac{-2}{5}\right)$$

$$\begin{array}{l} R_1 + \frac{1}{2}R_3 \\ R_2 - 2R_3 \end{array} \quad \begin{array}{c} x \quad y \quad z \\ \left[ \begin{array}{ccc|c} \textcircled{1} & 0 & 0 & 1 \\ 0 & \textcircled{1} & 0 & 4 \\ 0 & 0 & \textcircled{1} & -2 \end{array} \right] \end{array} \quad \text{RREF}$$

$$x = 1$$

$$y = 4$$

$$z = -2$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}$$

Ex: Solve by Gauss-Jordan Elimination

$$\begin{array}{c} x \quad y \quad z \\ \left[ \begin{array}{ccc|c} \textcircled{1} & 2 & 4 & -9 \\ 3 & 6 & 14 & -25 \\ 4 & 8 & 6 & -46 \end{array} \right] \end{array}$$

$$\begin{array}{l} R_2 - 3R_1 \\ R_3 - 4R_1 \end{array} \left[ \begin{array}{ccc|c} \textcircled{1} & 2 & 4 & -9 \\ 0 & 0 & \textcircled{2} & 2 \\ 0 & 0 & -10 & -10 \end{array} \right]$$

$$\frac{R_2}{2} \left[ \begin{array}{ccc|c} 1 & 2 & 4 & -9 \\ 0 & 0 & \textcircled{1} & 1 \\ 0 & 0 & -10 & -10 \end{array} \right]$$

$$\begin{array}{l} R_1 - 4R_2 \\ R_3 + 10R_2 \end{array} \begin{array}{c} x \quad y \quad z \\ \left[ \begin{array}{ccc|c} \textcircled{1} & 2 & 0 & -13 \\ 0 & 0 & \textcircled{1} & 1 \\ 0 & 0 & 0 & 0 \end{array} \right] \end{array}$$

RREF

- Circle the leading nonzero entry in each row.
- Any column without a circle gets a parameter.

$$\boxed{y = t}$$

$$x + 2y = -13 \rightarrow x = -13 - 2y \rightarrow \boxed{x = -13 - 2t}$$

$$\boxed{z = 1}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -13 \\ 0 \\ 1 \end{bmatrix} + t \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$$