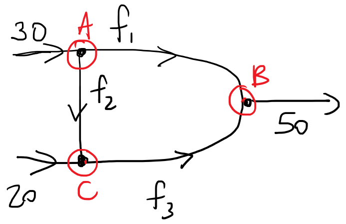
251DX01Feb10-1

February 10, 2021 11:27 AM

Section 2.4

Ex:



e.g. vehicles/hour

Assume flows cannot be negative.
Find all possible flows.

Inflow = outflow

$$\begin{cases} 30 = f_1 + f_2 \\ f_1 + f_3 = 50 \\ f_2 + 20 = f_3 \quad \rightarrow \quad f_2 - f_3 = -20 \end{cases}$$

$$\begin{array}{ccc} f_1 & f_2 & f_3 \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 30 \\ 1 & 0 & 1 & 50 \\ 0 & 1 & -1 & -20 \end{array}\right]$$

$$R_2 - R_1 \quad \left[\begin{array}{ccc|c} 1 & 1 & 0 & 30 \\ 0 & -1 & 1 & 20 \\ 0 & 1 & -1 & -20 \end{array}\right]$$

$$\frac{R_2}{(-1)} \quad \left[\begin{array}{ccc|c} 1 & 1 & 0 & 30 \\ 0 & 1 & -1 & -20 \\ 0 & 1 & -1 & -20 \end{array}\right]$$

$$\begin{array}{c} R_1 - R_2 \\ \\ R_3 - R_2 \end{array} \quad \begin{array}{c} \begin{array}{ccc} f_1 & f_2 & f_3 \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 1 & 50 \\ 0 & 1 & -1 & -20 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array} \quad \text{RREF}$$

$$\boxed{f_3 = t}$$

$$f_1 + f_3 = 50 \rightarrow \boxed{f_1 = 50 - t}$$

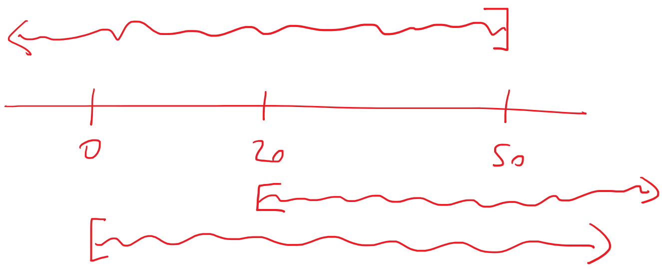
$$f_2 - f_3 = -20 \rightarrow \boxed{f_2 = -20 + t}$$

All flows must be non-negative:

$$f_3 = t \Rightarrow t \geq 0$$

AND $f_1 = 50 - t \Rightarrow 50 - t \geq 0 \Rightarrow 50 \geq t \Rightarrow t \leq 50$

AND $f_2 = -20 + t \Rightarrow -20 + t \geq 0 \Rightarrow t \geq 20$



$$20 \leq t \leq 50$$

$$\begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} = \begin{bmatrix} 50 \\ -20 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \qquad (20 \leq t \leq 50)$$

Ex: Set up a system, but don't solve

Balance

$v\, C_2H_2Cl_4 + w\, Ca(OH)_2 \rightarrow x\, C_2HCl_3 + y\, CaCl_2 + z\, H_2O$

Variables?

Equations (Restrictions / Constraints)

$$\begin{cases} C: & 2v = 2x \\ H: & 2v + 2w = x + 2z \\ Cl: & 4v = 3x + 2y \\ Ca: & w = y \\ O: & 2w = z \end{cases}$$

2.3 Linear Independence

Consider $c_1 \bar{u} + c_2 \bar{v} + c_3 \bar{w} = \bar{0}$

$c_1 = 0,\ c_2 = 0,\ c_3 = 0$ → $\bar{u}, \bar{v}, \bar{w}$ are linearly independent

$\{\bar{u}, \bar{v}, \bar{w}\}$ is "

$c_1 = 0,\ c_2 = 0,\ c_3 = 0$ and other solutions → dependent

$\{\bar{u}, \bar{v}, \bar{w}\}$ is "

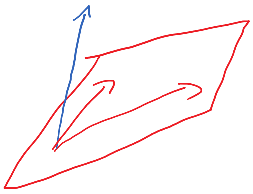
Ex: a) $\{\bar{u}, \bar{v}, \bar{w}\}$ is linearly independent

Is $\{\bar{u}, \bar{v}\}$ linearly independent?

YES

Given: $c_1\bar{u} + c_2\bar{v} + c_3\bar{w} = \bar{0} \Rightarrow c_1 = c_2 = c_3 = 0$

Conclude: $c_1\bar{u} + c_2\bar{v} = \bar{0} \Rightarrow c_1 = c_2 = 0$



b) $\{\bar{u}, \bar{v}, \bar{w}\}$ is linearly independent

Is $\{\bar{u}, \vec{v}, \bar{w}, \bar{x}\}$ linearly independent?

MAYBE

Ex A:

$\bar{u} = [1, 0, 0, 0]$

$\bar{v} = [0, 1, 0, 0]$

$\bar{w} = [0, 0, 1, 0]$

$\vec{x} = [0, 0, 0, 1]$

INDEPENDENT

Ex B:

$\vec{x} = 2\bar{u}$

$\{\bar{u}, \bar{v}, \bar{w}, 2\bar{u}\}$

DEPENDENT

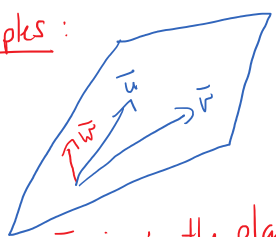
$2\bar{u} - \bar{x} = \bar{0}$

c) $\{\bar{u}, \bar{v}, \bar{w}\}$ is linearly dependent

Is $\{\bar{u}, \bar{v}, \bar{w}, \bar{x}\}$ linearly independent?

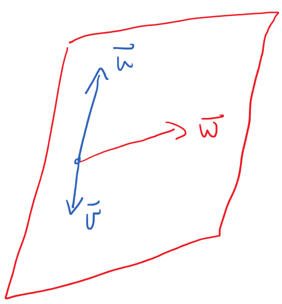
NO

Examples:



$\bar{w}$ is in the plane

$\bar{w} = -0.5\bar{v} + \bar{u}$



$\bar{u} = -2\bar{v}$

Span

Ex: Is $\begin{bmatrix}2\\3\\1\end{bmatrix}$ in $\underbrace{\text{span}\left(\begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}2\\5\\2\end{bmatrix}\right)}_{c_1\begin{bmatrix}1\\0\\1\end{bmatrix} + c_2\begin{bmatrix}2\\5\\2\end{bmatrix}}$?

Let $c_1\begin{bmatrix}1\\0\\1\end{bmatrix} + c_2\begin{bmatrix}2\\5\\2\end{bmatrix} = \begin{bmatrix}2\\3\\1\end{bmatrix}$

System is Solvable → YES

System has No Solution → NO

$$\begin{array}{cc|c} c_1 & c_2 & \\ \hline \end{array}$$

$$\left[\begin{array}{cc|c} 1 & 2 & 2 \\ 0 & 5 & 3 \\ 1 & 2 & 1 \end{array}\right]$$

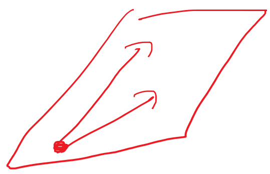
$\vdots$

$$\left[\begin{array}{cc|c} & & \\ & & \end{array}\right] \text{REF}$$

Ex. Describe span $\left(\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}4\\5\\6\end{bmatrix}\right)$

a) geometrically

plane through origin in $\mathbb{R}^3$ with $\vec{d}_1 = \begin{bmatrix}1\\2\\3\end{bmatrix}$, $\vec{d}_2 = \begin{bmatrix}4\\5\\6\end{bmatrix}$



b) algebraically

all are acceptable:

- Vector: $\vec{x} = \begin{bmatrix}0\\0\\0\end{bmatrix} + s\begin{bmatrix}1\\2\\3\end{bmatrix} + t\begin{bmatrix}4\\5\\6\end{bmatrix}$
- Parametric: ...
- $\vec{n} = \begin{bmatrix}1\\2\\3\end{bmatrix} \times \begin{bmatrix}4\\5\\6\end{bmatrix}$ $\quad \vec{p} = \begin{bmatrix}0\\0\\0\end{bmatrix}$
- Normal: $\vec{n} \cdot \vec{x} = \vec{n} \cdot \vec{p}$
- General: ...