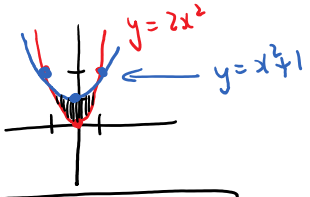
Week 6 Tuesday

October 8, 2019 8:58 AM

13.3 Area and Volume Cont'd

Ex: Area bounded by $y=2x^2$ and $y=x^2+1$?



Intersection

$y=y$

$2x^2=x^2+1$

$x^2=1$

$x=\pm 1$

$R: 2x^2 \le y \le x^2+1$

$-1 \le x \le 1$

$$A = \iint_R dA$$

$$= \int_{-1}^{1}\int_{2x^2}^{x^2+1} dy\,dx$$

$$= \int_{-1}^{1} \Big[y \Big]_{y=2x^2}^{y=x^2+1} dx$$

$$= \int_{-1}^{1} \left[x^2+1-(2x^2)\right] dx$$

$$= \int_{-1}^{1} (1-x^2)\,dx$$

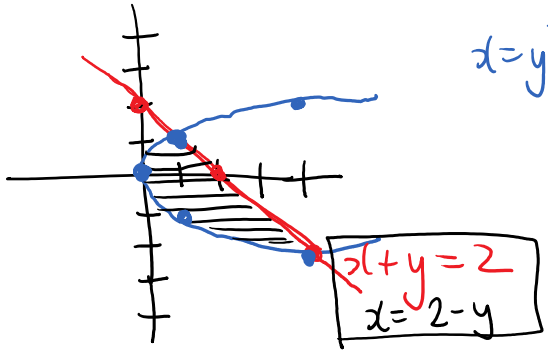
$$= \left[x - \frac{x^3}{3}\right]_{-1}^{1}$$

$$= \frac{2}{3} - \left(\frac{-2}{3}\right)$$

$$= \frac{4}{3}$$

Ex: Volume under $z=2+x$ over the region bounded by $x+y=2$ and $x=y^2$

R:



$x=y^2$

x	y
0	0
1	± 1
4	± 2

Horizontal Slices

R: $y^2 \le x \le 2-y$

$-2 \le y \le 1$

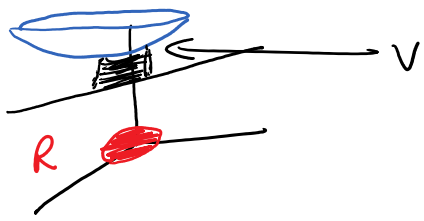
$$V = \iint_R f(x,y)\, dA$$

$$= \int_{-2}^{1}\int_{y^2}^{2-y} (2+x)\, dx\, dy$$

$$\vdots$$

$$= 16.2$$

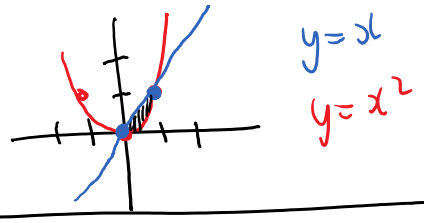
Volume Between 2 Surfaces



$$V = \iint_R (z_{top} - z_{bottom})\, dA$$

<u>Ex</u>: Volume between $z=4$, $z=x+1$, $y=x$ and $y=x^2$?

R:



$$\boxed{\begin{array}{ll} R: & x^2 \le y \le x \\ & 0 \le x \le 1 \end{array}}$$

Over this region $\quad 4 > x+1$

$z_{top} = 4$

$z_{bottom} = x+1$

$$V = \iint_R (z_{top} - z_{bottom})\, dA$$

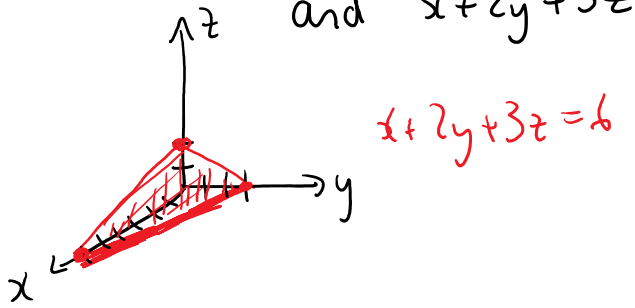
$$= \int_0^1 \int_{x^2}^{x} (4 - (x+1))\, dy\, dx$$

$$\vdots$$

$$= \frac{5}{12}$$

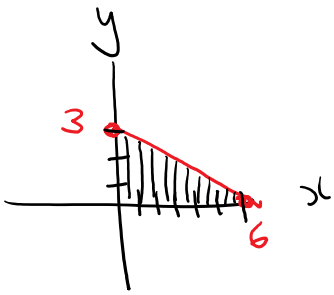
<u>Ex</u>: Set up double integrals for:

a) Volume bounded by $x=0, y=0, z=0$ and $x+2y+3z=6$



$R: \quad z=0 \rightarrow x+2y+3z=6$

$x+2y=6$



$$\boxed{\begin{aligned} x+2y&=6 \\ 2y&=6-x \\ y&=3-\tfrac{x}{2} \end{aligned}}$$

$$\boxed{\begin{aligned} R: \quad & 0 \le y \le 3-\tfrac{x}{2} \\ & 0 \le x \le 6 \end{aligned}}$$

$$V = \iint_R z \, dA$$

$$\boxed{\begin{aligned} x+2y+3z&=6 \\ 3z&=6-x-2y \\ z&=2-\tfrac{x}{3}-\tfrac{2y}{3} \end{aligned}}$$

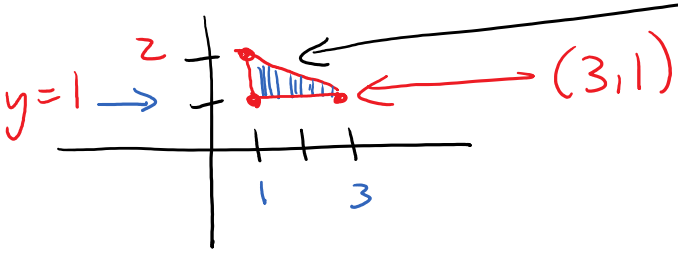
$$= \int_0^6 \int_0^{3-\frac{x}{2}} \left(2-\frac{x}{3}-\frac{2y}{3}\right) dy\, dx$$

$\vdots$

$$= 6$$

b) Volume bounded by $z=x$ and the triangle with vertices $(1,1,0)$, $(1,2,0)$ and $(3,1,0)$.

R: triangle



$y = mx + b$

$y = -\frac{1}{2}x + b$

Sub $(1,2)$: $2 = -\frac{1}{2} + b$

$b = \frac{5}{2}$

$y = -\frac{1}{2}x + \frac{5}{2}$

$$\boxed{R: \quad 1 \le y \le -\frac{1}{2}x + \frac{5}{2} \qquad 1 \le x \le 3}$$

$$V = \iint_R z \, dA$$

$$= \iint_R x \, dA$$

$$= \int_1^3 \int_1^{-\frac{x}{2}+\frac{5}{2}} x \, dy \, dx$$

$$\vdots$$

$$= \frac{5}{3}$$