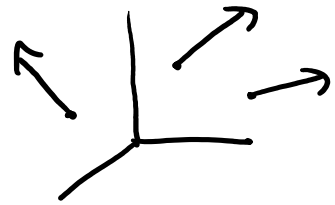
Ch 12 Differentiation

Ch 13 Integration

Ch 14 Vector Fields



e.g. fluid flows

(Need differentiation and integration)

13.1 Double Integrals

Ex: Evaluate $\int_0^{\pi}\int_0^{\pi}(xy+\sin x)\,dx\,dy$

Integrate w.r.t. x (y is a constant)

$$= \int_0^{\pi}\left[\frac{x^2y}{2} - \cos x\right]_{x=0}^{x=\pi} dy$$

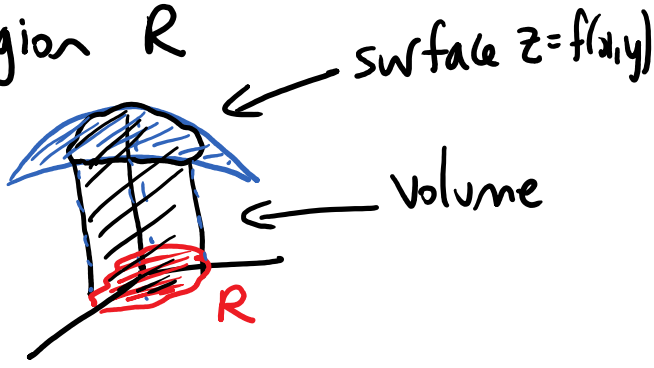
$$= \int_0^{\pi}\left[\frac{\pi^2 y}{2} + 1 - (0-1)\right] dy$$

$$= \int_0^{\pi} \left(\frac{\pi^2 y}{2} + 2 \right) dy$$

$$= \left[\frac{\pi^2 y^2}{4} + 2y \right]_{y=0}^{y=\pi}$$

$$= \frac{\pi^4}{4} + 2\pi$$

$\iint_R f(x,y)\, dx\, dy$ represents the volume under the surface $z = f(x,y)$ over the region R



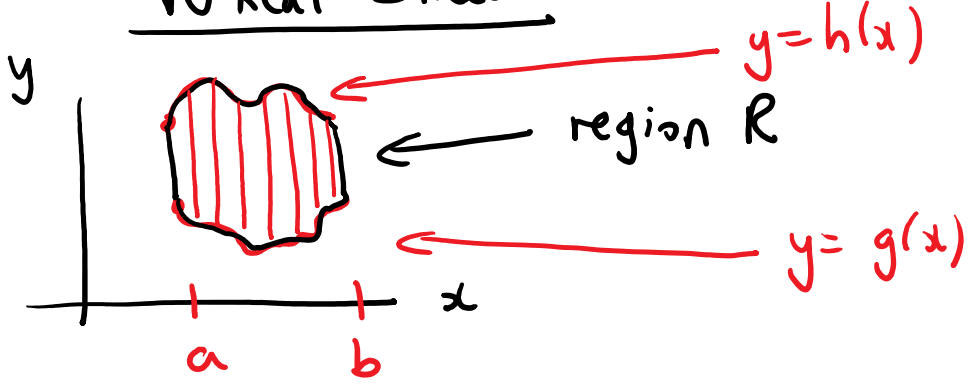
13.2 Double Integrals over General Regions

We'll consider $\iint_R f(x,y)\, dA$

dA: $dx\,dy$ or $dy\,dx$

Can use Vertical slices or horizontal slices to set up a double integral.

Vertical Slices



R: $g(x) \le y \le h(x)$

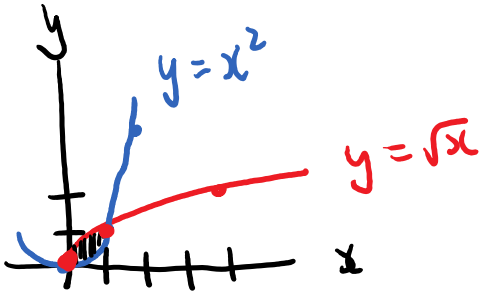
$a \le x \le b$

$$\iint_R f(x,y)\,dA = \int_a^b \int_{g(x)}^{h(x)} f(x,y)\,dy\,dx$$

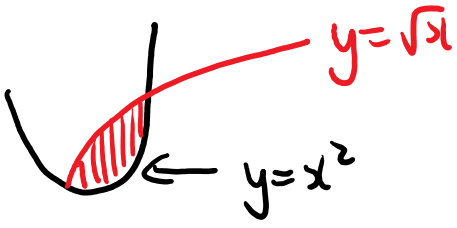
"Vertical Slices"

Ex: Evaluate $\iint_R xy\,dA$

R: bounded by $y = \sqrt{x}$ and $y = x^2$



$$R: \quad x^2 \le y \le \sqrt{x}$$
$$0 \le x \le 1$$



$$\text{Integral} = \int_0^1 \int_{x^2}^{\sqrt{x}} xy \, dy \, dx$$

$$= \int_0^1 \frac{xy^2}{2} \Big|_{y=x^2}^{y=\sqrt{x}} dx$$

$$= \int_0^1 \left[\frac{x \cdot x}{2} - \frac{x(x^2)^2}{2} \right] dx$$

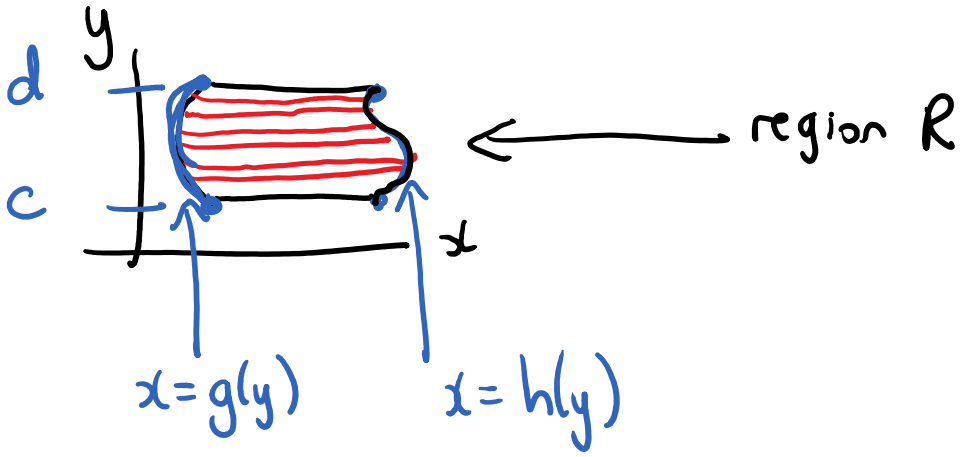
$$= \int_0^1 \left(\frac{x^2}{2} - \frac{x^5}{2} \right) dx$$

$$= \left[\frac{x^3}{6} - \frac{x^6}{12} \right]_0^1$$

$$= \frac{1}{6} - \frac{1}{12}$$

$$= \frac{1}{12}$$

Horizontal Slices



$$R: \quad g(y) \le x \le h(y), \quad c \le y \le d$$

$$\iint_R f(x,y)\,dA = \int_c^d \int_{g(y)}^{h(y)} f(x,y)\,dx\,dy$$

"Horizontal Slices"

Ex: Evaluate $\iint_R -y\,dA$

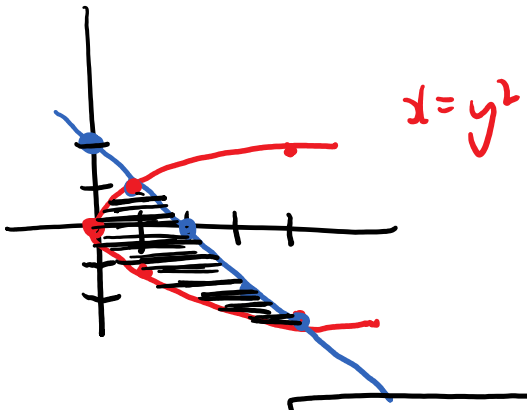
R: bounded by $x = y^2$ and $y = 2 - x$

$x = y^2$

y	x
0	0
± 1	1
± 2	4

$\rightarrow$

x	y
0	0
1	± 1
4	± 2



$y = 2 - x$

$x + y = 2$

$x = 2 - y$

R: $y^2 \leq x \leq 2 - y$

$-2 \leq y \leq 1$

$$\text{Integral} = \int_{-2}^{1} \int_{y^2}^{2-y} -y \, dx \, dy$$

"Horizontal Slices"

(Answer = $\frac{9}{4}$)