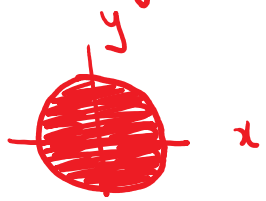
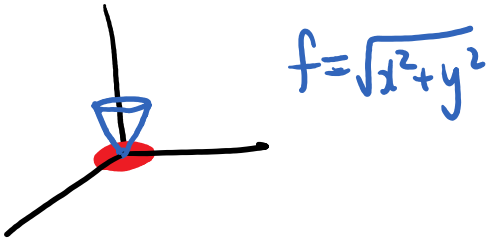
12.5 Cont'd

Ex: Find all absolute extrema of $f(x,y) = \sqrt{x^2+y^2}$ over the region $x^2+y^2 \leq 1$.

Domain $x^2+y^2 \leq 1$





1) interior critical points

$$f = \sqrt{x^2+y^2}$$

$$f_x = \tfrac{1}{2}(x^2+y^2)^{-1/2}(2x) = \frac{x}{\sqrt{x^2+y^2}}$$

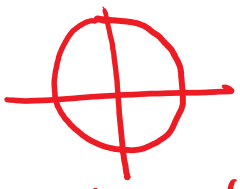
$$f_y = \tfrac{1}{2}(x^2+y^2)^{-1/2}(2y) = \frac{y}{\sqrt{x^2+y^2}}$$

both 0 or undefined?

$\Rightarrow (x,y) = (0,0)$

$f(0,0) = \sqrt{0^2+0^2} = 0$

2) Evaluate f at corners and critical points on sides of boundary



Boundary of Domain $x^2+y^2=1$

Note: A circle has no corners and 1 side

Side 1: $x^2+y^2=1$

$y^2=1-x^2$

$y=\pm\sqrt{1-x^2}$ $\rightarrow$ $f=\sqrt{x^2+y^2}$

$f=1$

Let $f'=0$: $f'=0$ everywhere on $x^2+y^2=1$

Every point on Side 1 is a critical point.

SUMMARY

(x,y)	f
$(0,0)$	0
$\{(x,y) \mid x^2+y^2=1\}$	1

f has an absolute max of 1 on $x^2+y^2=1$

" min of 0 at $(x,y)=(0,0)$

12.6 Increments and Linear Approximation

Error estimates

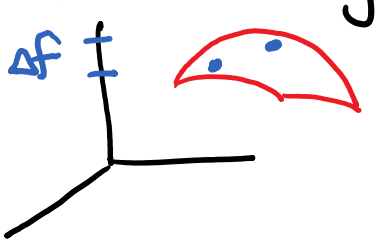
Differential of f: $\boxed{df = f_x\,dx + f_y\,dy}$

Ex: $f = x^2y^3 + 3x$

$df = (2xy^3 + 3)dx + (3x^2y^2)dy$

Surface $z = f(x,y)$

Δf = actual change in f between 2 points



Ex: $f = x^2y^3 + 3x$

Find Δf as we go from $(1,1)$ to $(0.9, 1.2)$

$$\Delta f = f(0.9, 1.2) - f(1,1)$$
$$= 4.09968 - 4$$
$$= 0.09968$$

dx, dy represent change in x and y

$\Delta f \approx df$ when dx, dy are small

Linear Approximation

$f_{new} = f_{old} + \underbrace{\text{change in } f}_{\Delta f}$

$$f(x+dx, y+dy) \approx f(x,y) + df$$

Linear Approximation formula

Ex: $f = \sqrt{x}\sqrt{y}$

Approximate $f(4.1, 9.3)$

$x = 4$
$y = 9$
} "nice values" near $(x,y) = (4.1, 9.3)$

$dx = 0.1$ $\quad 4 \to 4.1$

$dy = 0.3$ $\quad 9 \to 9.3$

$$df = f_x\,dx + f_y\,dy$$

$$= \frac{\sqrt{y}}{2\sqrt{x}}\,dx + \frac{\sqrt{x}}{2\sqrt{y}}\,dy$$

$$= \frac{3}{4}(0.1) + \frac{2}{6}(0.3)$$

$$= 0.175$$

Linear Approximation formula

$$f(x+dx, y+dy) \approx f(x,y) + df$$

$$f(4.1, 9.3) \approx f(4,9) + df$$

$$\approx 6 + 0.175$$

$$\approx 6.175$$