1

R: $\sqrt{x} \le y \le 3$
$0 \le x \le 9$ (Vertical Slices)

R: $0 \le x \le y^2$
$0 \le y \le 3$ (Horizontal Slices)

$$\text{Integral} = \int_0^3 \int_0^{y^2} \sqrt{1+y^3}\, dx\, dy$$

$$= \int_0^3 x\sqrt{1+y^3}\,\Big|_{x=0}^{x=y^2} dy$$

$$= \int_0^3 y^2\sqrt{1+y^3}\, dy$$

$u = 1+y^3$
$du = 3y^2\, dy$
$\frac{du}{3} = y^2\, dy$

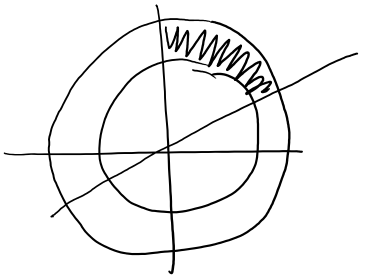
$\text{Integral} = \int \frac{\sqrt{u}\, du}{3}$
$= \frac{1}{3}\frac{2}{3}u^{3/2} + C$

$$= \frac{2}{}(1+u^3)^{3/2}\Big|^3$$

$$= \frac{2}{9}\left(1+y^3\right)^{3/2}\Big|_0^3$$

$$= \frac{2}{9}\left[28^{3/2} - 1\right]$$

2 a)



$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$
$$= \tan^{-1}\left(\frac{x}{\sqrt{3}} \cdot \frac{1}{x}\right)$$
$$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$
$$= \frac{\pi}{6}$$

R: $2 \le r \le \sqrt{5}$

$\frac{\pi}{6} \le \theta \le \frac{\pi}{2}$

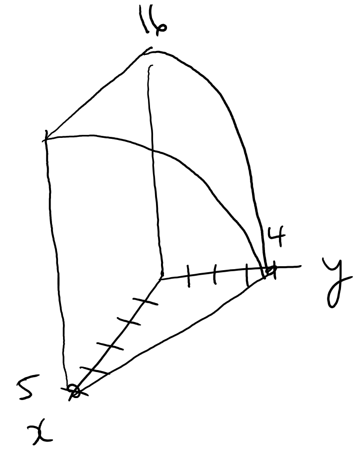
$$A = \int_{\pi/6}^{\pi/2} \int_{2}^{\sqrt{5}} r\,dr\,d\theta$$

b)

$$m = \iint_R \delta\, dA$$

$$= \int_{\pi/6}^{\pi/2} \int_{2}^{\sqrt{5}} (r\cos\theta)\, r\,dr\,d\theta$$

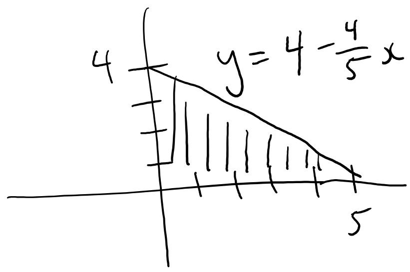
3



$0 \le z \le 16 - y^2$

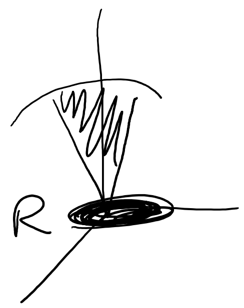
$0 \le y \le 4 - \frac{4}{5}x$

$0 \le x \le 5$



$$V = \int_0^5 \int_0^{4-\frac{4}{5}x} \int_0^{16-y^2} dz\,dy\,dx$$

4



R: $z = z$

$2r = \sqrt{45 - r^2}$

$4r^2 = 45 - r^2$

$5r^2 = 45$

$r^2 = 9$

$\cancel{r = \pm 3}$ $r = 3$

$0 \le r \le 3$

$0 \le \theta \le 2\pi$

$$V = \iint_R \left(z_{top} - z_{bottom}\right) dA$$

$$= \int_0^{2\pi} \int_0^3 \left(\sqrt{45 - r^2} - 2r\right) r\,dr\,d\theta$$

$$= \int_0^{2\pi} \int_0^3 \left(r\sqrt{45 - r^2} - 2r^2\right) dr\,d\theta$$

$$\boxed{\begin{aligned} &\text{Sub } u = 45 - r^2 \\ &du = -2r\,dr \\ &-\tfrac{1}{2}\,du = r\,dr \\ &\int r\sqrt{45 - r^2}\,dr = \int -\tfrac{1}{2}\sqrt{u}\,du \\ &= -\tfrac{1}{3}u^{3/2} + C \end{aligned}}$$

$$= \int_0^{2\pi} \left[-\frac{1}{3}\left(45 - r^2\right)^{3/2} - \frac{2r^3}{3} \right]_0^3 d\theta$$

$$= \int_0^{2\pi} \left[-\frac{1}{3}(36)^{3/2} - \frac{54}{3} + \frac{1}{3}(45)^{3/2} \right] d\theta$$

$$= \frac{2\pi}{3}\left[45^{3/2} - 270 \right]$$