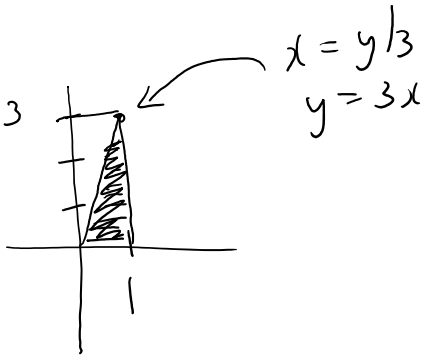
(1) R: $\frac{y}{3} \le x \le 1$, $0 \le y \le 3$ (Horizontal Slices)



R: $0 \le y \le 3x$, $0 \le x \le 1$ (Vertical Slices)

$$\text{Integral} = \int_0^1 \int_0^{3x} \sin x^2 \, dy \, dx$$

$$= \int_0^1 y \sin x^2 \Big|_{y=0}^{y=3x} dx$$

$$= \int_0^1 3x \sin x^2 \, dx$$

$u = x^2$

$du = 2x\,dx$

$\frac{3}{2} du = 3x\,dx$

$\text{Integral} = \int \frac{3}{2} \sin u \, du$

$= -\frac{3}{2} \cos u + C$

$1 \quad x=1$

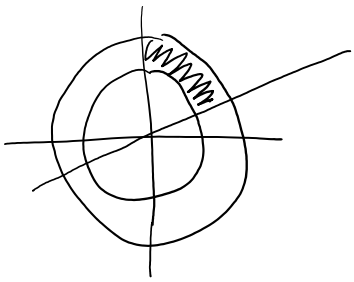
$= \frac{3}{2}\cos 1$

$= -\frac{3}{2}\cos x^2 \Big|_{x=0}^{x=1}$

$= -\frac{3}{2}\cos 1 + \frac{3}{2}$

or $\frac{3}{2}(1-\cos 1)$

(2) a)



$y = \frac{x}{\sqrt{3}}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$= \tan^{-1}\left(\frac{x}{\sqrt{3}} \cdot \frac{1}{x}\right)$

$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$= \frac{\pi}{6}$

$R: \quad 3 \le r \le \sqrt{10}$

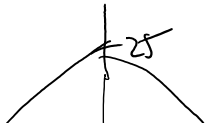
$\frac{\pi}{6} \le \theta \le \frac{\pi}{2}$

$$A = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \int_{3}^{\sqrt{10}} r \, dr \, d\theta$$

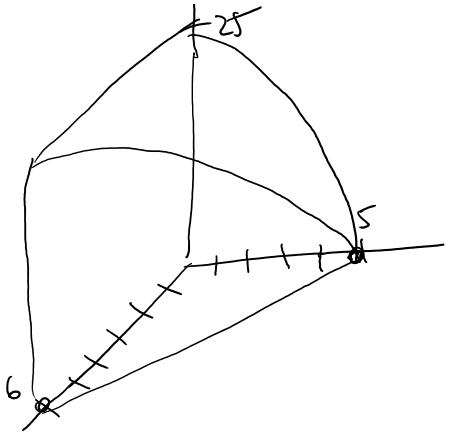
b) $$m = \iint_R \delta \, dA$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \int_{3}^{\sqrt{10}} (r\cos\theta)^2 \, r \, dr \, d\theta$$

(3)



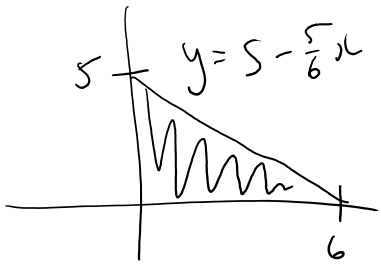
(3)



$0 \le z \le 25 - y^2$

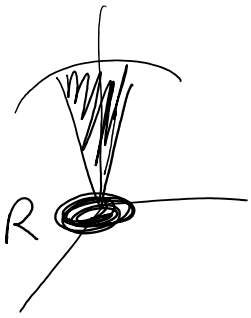
$0 \le y \le 5 - \frac{5}{6}x$

$0 \le x \le 6$



$$V = \int_0^6 \int_0^{5-\frac{5}{6}x} \int_0^{25-y^2} dz\, dy\, dx$$

(4)



R: $z = z$

$2r = \sqrt{20 - r^2}$

$4r^2 = 20 - r^2$

$5r^2 = 20$

$r^2 = 4$

~~$r = \pm 2$~~ $r = 2$

$0 \le r \le 2$

$0 \le \theta \le 2\pi$

$$V = \iint_R (z_{top} - z_{bottom})\, dA$$

$$= \int_0^{2\pi} \int_0^2 \left(\sqrt{20-r^2} - 2r\right) r\, dr\, d\theta$$

$$= \int_0^{2\pi} \int_0^2 \left(r\sqrt{20-r^2} - 2r^2\right) dr\, d\theta$$

$u = 20 - r^2$

$du = -2r\, dr$

$-\frac{1}{2} du = r\, dr$

$\int r\sqrt{20-r^2}\, dr = \int -\frac{1}{2}\sqrt{u}\, du$

$= -\frac{1}{3} u^{3/2} + C$

$$= \int_0^{2\pi} \left[-\frac{1}{3}(20-r^2)^{3/2} - \frac{2r^3}{3} \right]_0^2 d\theta$$

$$= \int_0^{2\pi} \left[-\frac{1}{3}(16)^{3/2} - \frac{16}{3} + \frac{20}{3}^{3/2} \right] d\theta$$

$$= \frac{2\pi}{3}\left[20^{3/2} - 80 \right]$$