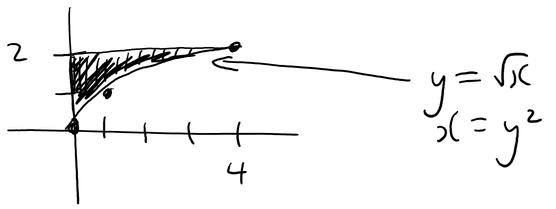
250B-t3sol-DX01

September 29, 2020 11:02 AM

(1) $R: \quad \sqrt{x} \le y \le 2$
$\quad\quad 0 \le x \le 4$ (Vertical Slices)



$R: \quad 0 \le x \le y^2$
$\quad\quad 0 \le y \le 2$ (Horizontal Slices)

$$\text{Integral} = \int_0^2 \int_0^{y^2} x\sqrt{1+y^5}\, dx\, dy$$

$$= \int_0^2 \frac{x^2}{2}\sqrt{1+y^5}\,\Bigg|_{x=0}^{x=y^2} dy$$

$$= \int_0^2 \frac{y^4}{2}\sqrt{1+y^5}\, dy$$

$$u = 1+y^5$$
$$du = 5y^4\,dy$$
$$\frac{du}{10} = \frac{y^4\,dy}{2}$$
$$\text{Integral} = \int \frac{u^{1/2}\,du}{10}$$
$$= \frac{1}{10}\cdot\frac{2}{3}u^{3/2} + C$$

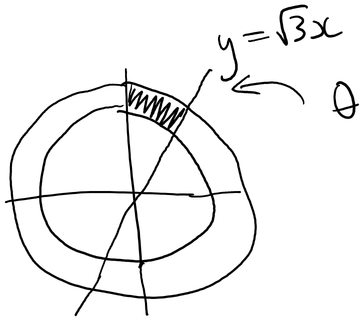
$$= \frac{1}{\ }\left(1+y^5\right)^{3/2}\Big|^{2}$$

$$= \frac{1}{15}(1+y^5)^{3/2} \Big|_0^2$$

$$= \frac{1}{15}\left[33^{3/2} - 1 \right]$$

(2)

a)



$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$
$$= \tan^{-1}\left(\frac{\sqrt{3}x}{x}\right)$$
$$= \tan^{-1}(\sqrt{3})$$
$$= \frac{\pi}{3}$$

R: $\sqrt{8} \le r \le 3$

$\frac{\pi}{3} \le \theta \le \frac{\pi}{2}$

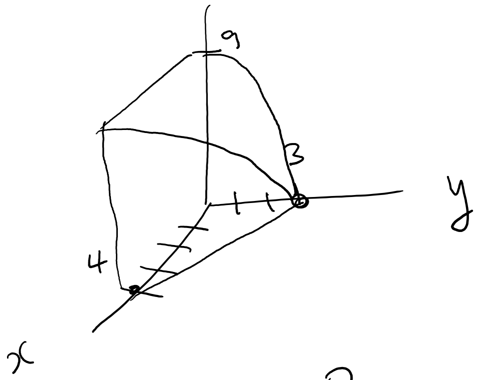
$$A = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \int_{\sqrt{8}}^{3} r \, dr \, d\theta$$

b)

$$m = \iint_R \delta \, dA$$

$$= \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \int_{\sqrt{8}}^{3} (r\sin\theta) \, r \, dr \, d\theta$$

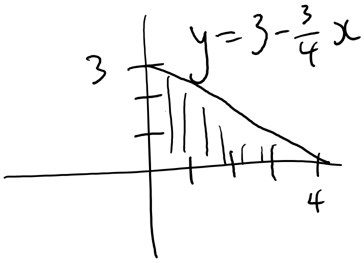
③



R: $0 \le z \le 9-y^2$

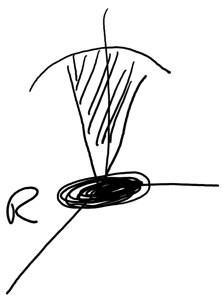
$0 \le y \le 3 - \frac{3}{4}x$

$0 \le x \le 4$



$$V = \int_0^4 \int_0^{3-\frac{3}{4}x} \int_0^{9-y^2} dz\,dy\,dx$$

④



R: $z = z$

$2r = \sqrt{80 - r^2}$

$4r^2 = 80 - r^2$

$5r^2 = 80$

$r^2 = 16$

$\cancel{r = \pm 4}$ $r = 4$

$0 \le r \le 4$

$0 \le \theta \le 2\pi$

$$V = \iint_R \left(z_{top} - z_{bottom}\right) dA$$

$$= \int_0^{2\pi} \int_0^4 \left(\sqrt{80 - r^2} - 2r\right) r\, dr\, d\theta$$

$$= \int_0^{2\pi} \int_0^4 \left(r\sqrt{80 - r^2} - 2r^2\right) dr\, d\theta$$

$$\boxed{\begin{aligned} \text{Let } u &= 80 - r^2 \\ du &= -2r\,dr \\ -\tfrac{1}{2}\, du &= r\,dr \\ \int r\sqrt{80-r^2}\, dr &= \int -\tfrac{1}{2}\sqrt{u}\, du \\ &= -\tfrac{1}{3} u^{3/2} + C \end{aligned}}$$

$$= \int_0^{2\pi} \left[-\frac{1}{3}\left(80 - r^2\right)^{3/2} - \frac{2r^3}{3}\right]_0^4 d\theta$$

$$= \int_0^{2\pi} \left[-\frac{1}{3}(64)^{3/2} - \frac{128}{3} + \frac{1}{3}(80)^{3/2}\right] d\theta$$

$$= \frac{2\pi}{3}\left[80^{3/2} - 640\right]$$