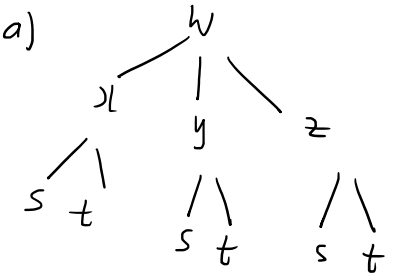
(1) a)



$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial t}$$

$$= (y+z)(-2t) + (x+z)(2t) + (x+y)\frac{1}{2\sqrt{s}\sqrt{t}}$$

b) $s=1, t=2 \rightarrow \quad x=-3 \quad y=5 \quad z=\sqrt{2}$

$$\frac{\partial w}{\partial t} = (5+\sqrt{2})(-4) + (-3+\sqrt{2})(4) + 2\,\frac{1}{2\sqrt{2}}$$

$$= -32 + \frac{1}{\sqrt{2}}$$

(2) a) $\vec{u} = \dfrac{[7,8]}{\|[7,8]\|} = \dfrac{1}{\sqrt{113}}[7,8]$

$$\nabla f = [-0.2x, 0.4y]$$

$$\nabla f(3,4) = [-0.6, 1.6]$$

$$D_{\vec{u}} f = \nabla f \cdot \vec{u}$$

$$= \frac{8.6}{\sqrt{113}} \quad \frac{^\circ C}{Km}$$

b) $\frac{8.6}{\sqrt{113}}(10) = \frac{86}{\sqrt{113}}$ $\frac{^\circ C}{h}$

c) The maximum rate of increase of f is $\|\nabla f\| = \|[-0.6, 1.6]\| = \sqrt{2.92}$ $\frac{^\circ C}{km}$

(3) $f_x = \frac{2}{x} - 10$

$f_y = \frac{1}{y^2} - 4$

} both 0 or undefined

Since $x > 0$ and $y > 0$, neither will be undefined

$\frac{2}{x} - 10 = 0 \rightarrow \frac{2}{x} = 10 \rightarrow x = \frac{1}{5}$

$\frac{1}{y^2} - 4 = 0 \rightarrow \frac{1}{y^2} = 4 \rightarrow y^2 = \frac{1}{4} \rightarrow y = \pm\frac{1}{2} \rightarrow y = \frac{1}{2}$

The critical point is $(\frac{1}{5}, \frac{1}{2})$

b)

Point	$f_{xx} = \frac{-2}{x^2}$	$f_{xy} = 0$	$f_{yy} = -\frac{2}{y^3}$	$\Delta = f_{xx}f_{yy} - (f_{xy})^2$
$(\frac{1}{5}, \frac{1}{2})$	-50	0	-16	>0

Since $\Delta > 0$ and $f_{xx} < 0$,
the point is a local maximum.

(4) $f = (x-4)^2 + (y+2)^2 + (z-6)^2$

$g = 2x + 5y + 3z$

$\nabla f = \lambda \nabla g$

$[2(x-4), 2(y+2), 2(z-6)] = \lambda [2, 5, 3]$

$2(x-4) = 2\lambda \rightarrow \lambda = x-4$

$2(y+2) = 5\lambda \rightarrow \lambda = \frac{2}{5}(y+2)$

$2(z-6) = 3\lambda \rightarrow \lambda = \frac{2}{3}(z-6)$

Conclude $x-4 = \frac{2}{5}(y+2) = \frac{2}{3}(z-6)$

$\frac{5}{2}(x-4) = y+2$

$\boxed{y = \frac{5}{2}(x-4) - 2}$

$\frac{3}{2}(x-4) = z-6$

$\boxed{z = \frac{3}{2}(x-4) + 6}$

$$\left.\begin{aligned} y &= \tfrac{5}{2}(x-4) - 2 \\ z &= \tfrac{3}{2}(x-4) + 6 \end{aligned}\right\} \rightarrow \quad 2x + 5y + 3z = 14$$

$$2x + \frac{25}{2}(x-4) - 10 + \frac{9}{2}(x-4) + 18 = 14$$

$$19x = 74$$

$$x = \frac{74}{19}$$

$x = \frac{74}{19} \rightarrow y = \frac{-43}{19}$ and $z = \frac{111}{19} \quad \rightarrow f = \frac{2}{19}$