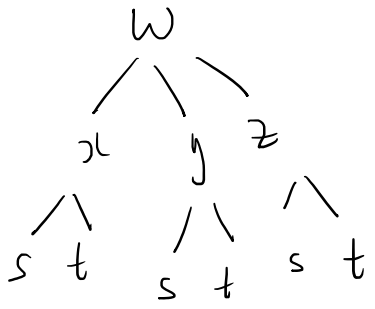
(1) a)



$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial t}$$

$$= yz(2t) + xz(-2t) + xy\left[2(2s+3t)(3)\right]$$

b) $s=1 \quad t=2 \rightarrow \quad x=5 \quad y=-3 \quad z=64$

$$\frac{\partial w}{\partial t} = -3(64)(4) + 5(64)(-4) - 15(6)(8)$$

$$= -2768$$

(2) a) $\vec{u} = \dfrac{[6,7]}{\|[6,7]\|} = \dfrac{1}{\sqrt{85}}[6,7]$

$\nabla f = [-0.2x, 0.4y]$

$\nabla f(3,4) = [-0.6, 1.6]$

$D_{\vec{u}} f = \nabla f \cdot \vec{u}$

$= \dfrac{7.6}{\sqrt{85}} \dfrac{^\circ C}{km}$

b) $\dfrac{7.6}{\sqrt{85}}(5) = \dfrac{38}{\sqrt{85}} \dfrac{^\circ C}{h}$

c) In the direction of ∇f,
that is $[-0.6, 1.6]$

(3) a) $f_x = \dfrac{-3}{x} + 12$

$f_y = \dfrac{-1}{y^2} + 9$

} both 0 or undefined

Since $x > 0$ and $y > 0$, neither is undefined.

$-\dfrac{3}{x} + 12 = 0 \rightarrow -\dfrac{3}{x} = -12 \rightarrow x = \dfrac{1}{4}$

$-\dfrac{1}{y^2} + 9 = 0 \rightarrow \dfrac{1}{y^2} = 9 \rightarrow y = \pm\dfrac{1}{3} \rightarrow y = \dfrac{1}{3}$

The critical point is $\left(\frac{1}{4}, \frac{1}{3}\right)$

b)

POINT	$f_{xx} = \frac{3}{x^2}$	$f_{xy} = 0$	$f_{yy} = \frac{2}{y^3}$	$\Delta = f_{xx}f_{yy} - (f_{xy})^2$
$\left(\frac{1}{4}, \frac{1}{3}\right)$	48	0	54	> 0

Since Δ and $f_{xx} > 0$,
the point is a local minimum.

(4) $f = (x-5)^2 + (y+6)^2 + (z-8)^2$

$g = 2x + 3y + 5z$

$\nabla f = \lambda \nabla g$

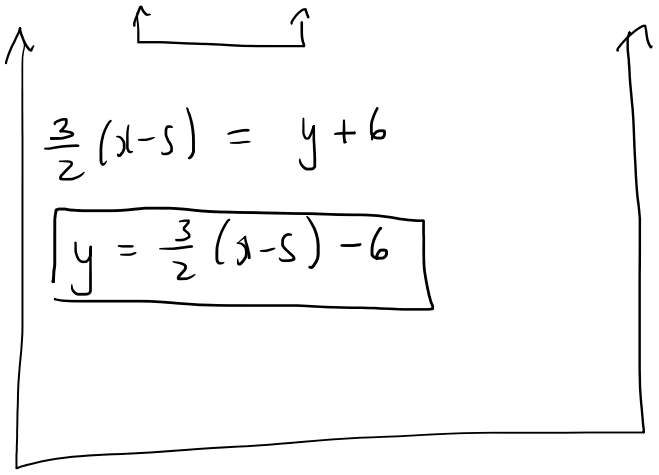
$[2(x-5), 2(y+6), 2(z-8)] = \lambda [2, 3, 5]$

$2(x-5) = 2\lambda \rightarrow \lambda = x-5$

$2(y+6) = 3\lambda \rightarrow \lambda = \frac{2}{3}(y+6)$

$2(z-8) = 5\lambda \rightarrow \lambda = \frac{2}{5}(z-8)$

Conclude $x-5 = \frac{2}{3}(y+6) = \frac{2}{5}(z-8)$



$\frac{3}{2}(x-5) = y+6$

$\boxed{y = \frac{3}{2}(x-5)-6}$

$\frac{5}{2}(x-5) = z-8$

$\boxed{z = \frac{5}{2}(x-5)+8}$

$$\left.\begin{aligned} y &= \tfrac{3}{2}(x-5)-6 \\ z &= \tfrac{5}{2}(x-5)+8 \end{aligned}\right\} \rightarrow \quad 2x+3y+5z = 12$$

$$2x + \tfrac{9}{2}(x-5) - 18 + \tfrac{25}{2}(x-5) + 40 = 12$$

$$19x = 75$$

$$x = \tfrac{75}{19}$$

$x = \frac{75}{19} \rightarrow \quad y = \frac{-144}{19}$ and $z = \frac{102}{19} \quad \rightarrow f = \frac{200}{19}$