Math250B-DX02 T1 Solutions

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① $dV = V_r\,dr + V_h\,dh$

$dV = \frac{2\pi}{3} rh\,dr + \frac{\pi}{3} r^2\,dh$

$$\boxed{\begin{array}{ll} r = 0.5\,m & dr = 0.02\,m \\ h = 2\,m & dh = -0.05\,m \end{array}}$$

$dV = \frac{2\pi}{3}(0.5)(2)(0.02) + \frac{\pi}{3}(0.5)^2(-0.05)$

$\simeq 0.03\ m^3$

② $z_x = 2xy^3 - y = -50$

$z_y = 3x^2y^2 - x = 111$

$\vec{n} = [-z_x, -z_y, 1] = [50, -111, 1]$

$x = -3,\ y = 2 \Rightarrow z = 78$

$\vec{n} \cdot \vec{x} = \vec{n} \cdot \vec{p}$

$$\begin{bmatrix} 50 \\ -111 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 50 \\ -111 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ 2 \\ 78 \end{bmatrix}$$

$50x - 111y + z = -294$

③ $f_y = -x^4 \sin y + 2x\cos(xy) + 7e^{7y}$

$f_{yx} = -4x^3 \sin y + \underbrace{2\cos(xy) - 2xy\sin(xy)}_{\text{Product Rule}}$

④ 1) Interior Critical Points

$$\left.\begin{aligned} z_x &= 2y^2 \\ z_y &= 4xy \end{aligned}\right\} \text{ both } 0 \text{ or undefined}$$

$\Rightarrow y = 0$

(x can be any value in $-\sqrt{21} < x < \sqrt{21}$)

$\boxed{\text{Critical Points : } (x, 0)}$

NOTE : A circle has 1 side and 0 corners

2) Critical Points on side : $x^2 + y^2 = 21$

$$z = 2xy^2$$

$$z = 2x(21 - x^2)$$

$$z = 2(21x - x^3)$$

Set $z' = 0$: $\quad z' = 2(21 - 3x^2) = 0$

$$21 - 3x^2 = 0$$

$$x^2 = 7$$

$$x = \pm\sqrt{7}$$

$$x^2 = 7 \Rightarrow y^2 = 14 \Rightarrow y = \pm\sqrt{14}$$

$$\boxed{\text{Critical Points: } (\pm\sqrt{7}, \pm\sqrt{14})}$$

Point	$z = 2xy^2$
$(x, 0)$	0
$(\sqrt{7}, \pm\sqrt{14})$	$28\sqrt{7}$
$(-\sqrt{7}, \pm\sqrt{14})$	$-28\sqrt{7}$

Answer: The maximum value is $28\sqrt{7}$, achieved at $(\sqrt{7}, \pm\sqrt{14})$