Math250B-DX01 T1 Solutions

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(1)

$$dA = A_b\,db + A_\theta\,d\theta$$

$$= b\sin\theta\,db + \frac{b^2}{2}\cos\theta\,d\theta$$

$$\boxed{\begin{array}{ll} b = 300 & db = 7 \\ \theta = 45^\circ & d\theta = -3^\circ = -\frac{\pi}{60}\text{ rad} \end{array}}$$

$$dA = 300(\sin 45^\circ)(7) + \frac{300^2}{2}(\cos 45^\circ)\left(\frac{-\pi}{60}\right)$$

$$\approx -181\text{ m}^2$$

(2)

$$f_y = -x^3\sin y + 4xe^{4xy} + \frac{2y}{y^2+2}$$

$$f_{yx} = -3x^2\sin y + \underbrace{4e^{4xy} + 16xye^{4xy}}_{\text{Product Rule}}$$

(3)

$$z_x = y - 14x = 44$$

$$z_y = x + 9y^2 = 33$$

$$\vec{n} = [-z_x, -z_y, 1]$$

$$= [-44, -33, 1]$$

$x = -3,\ y = 2 \Rightarrow z = -45$

$$\vec{n} \cdot \vec{x} = \vec{n} \cdot \vec{p}$$

$$\begin{bmatrix} -44 \\ -33 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -44 \\ -33 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} -3 \\ 2 \\ -45 \end{bmatrix}$$

$$-44x - 33y + z = 21$$

4 1) Interior Critical Points

$$\left.\begin{array}{l} z_x = 2y^2 \\ z_y = 4xy \end{array}\right\} \text{both } 0 \text{ or undefined}$$

$\Rightarrow y = 0$

(x can be any real number in $-\sqrt{15} < x < \sqrt{15}$)

$$\boxed{\text{Critical Points : } (x, 0)}$$

NOTE: A circle has 1 side and 0 corners

2) Critical Points on side : $x^2 + y^2 = 15$

$$z = 2xy^2$$

$$z = 2x(15 - x^2)$$

$$z = 2(15x - x^3)$$

Set $z' = 0$: $z' = 2(15 - 3x^2) = 0$

$$15 - 3x^2 = 0$$

$$x^2 = 5$$

$$x = \pm\sqrt{5}$$

$$x^2 = 5 \Rightarrow y^2 = 10 \Rightarrow y = \pm\sqrt{10}$$

$$\boxed{\text{Critical Points : } (\pm\sqrt{5}, \pm\sqrt{10})}$$

Point	$z = 2xy^2$
$(x, 0)$	0
$(\sqrt{5}, \pm\sqrt{10})$	$20\sqrt{5}$
$(-\sqrt{5}, \pm\sqrt{10})$	$-20\sqrt{5}$

Answer: The maximum value is $20\sqrt{5}$, achieved at $(x, y) = (\sqrt{5}, \pm\sqrt{10})$