MATH 250B REVIEW PROBLEMS

12.4 1. Find the equation of the plane tangent to $z = y^3 \ln x$ at the point $(e, 2, 8)$.

12.5 2. Suppose the temperature T on the disk $x^2 + y^2 \leq 1$ is given by $T = 2x^2 + y^2 - y$. Find the hottest and coldest points on the disk and the temperature at these points.

12.6 3. An object's mass m, kinetic energy E and speed v are related by $m = \frac{2E}{v^2}$. Find the maximum relative error in the object's mass given that the relative errors for the kinetic energy and speed are within $\pm 2\%$ and $\pm 3\%$ respectively.

12.7 4. Let $f = x^4 - y^4$ where $x = u^2 + v^2$ and $y = u + 2$. Find $\frac{\partial f}{\partial u}$ when $(u, v) = (1, -1)$.

12.7 5. Find $\frac{\partial z}{\partial y}$ given $3x^2z - x^2y^2 + 2z^3 + 3yz = 0$

12.8 6. The temperature (in $^\circ$ C) on the surface of a metal plate is $T = 20 - 4x^2 - y^2$, where x and y are in cm. In which direction from $(2, -3)$ does the temperature increase most rapidly? What is the rate of increase in this direction?

12.8 7. Over a certain region of space the electric potential is $\phi = 5x^2 - 3xy + xyz$, where ϕ is measured in V and position is in m. Find the rate of change of the potential at the point $(3, 4, 5)$ in the direction $\mathbf{i} + \mathbf{j} - \mathbf{k}$.

12.8 8. Find the equation of the tangent plane to $x^2 - 2y^2 + z^2 + yz = 2$ at the point $(2, 1, -1)$.

12.9 9. Use Lagrange Multipliers to find the points on the surface $z = 4xy + 3$ that are closest to the origin.

12.10 10. Find and classify all critical points on the surface $z = 3x^2y + y^3 - 3x^2 - 3y^2$

13.2 11. Evaluate $\int_0^2 \int_{3x}^6 e^{-y^2}\, dy\, dx$

12. Set up double integrals for the following:

13.3 a) The area between $y = 4 - x^2$ and $y = 3$

b) The volume between $z = 4$ and $z = 2 + x$, over the region bounded by $y = 4 - x^2$ and $y = 3$

13.4 13. Evaluate $\int_0^3 \int_0^{\sqrt{9-x^2}} \sin(x^2 + y^2)\, dy\, dx$

13.4 14. Find the area bounded by the circles $r = 1$ and $r = 2\sin\theta$

15. Set up polar double integrals that give the volumes of the following solids:

13.4 a) The solid bounded by $z = 32 - x^2 - y^2$ and $z = x^2 + y^2$

b) The part of the ball $x^2+y^2+z^2 \leq 9$ in the first octant and inside the cylinder $x^2+y^2 = 3y$

13.5 16. Let $\delta = y$. Set up a polar double integral giving the mass of the region bounded by $2 \leq x^2 + y^2 \leq 3, x \geq 0$ and $y \geq \sqrt{3}x$.

13.6 17. Set up a triple integral in rectangular coordinates for the volume between $x^2+y^2+z^2 = 12$ and $z = x^2 + y^2$.

13.7 18. Set up a triple integral in cylindrical coordinates for $\iiint\limits_Q \sqrt{x^2+y^2}\, dV$ where Q is the region in the first octant inside $z = \sqrt{25 - x^2 - y^2}$ and outside $x^2 + y^2 = 4x$.

13.7 19. Use a triple integral in cylindrical coordinates to find the volume of the solid region inside both the cylinder $x^2 + y^2 = 2y$ and the sphere $x^2 + y^2 + z^2 = 4$.

13.7 20. Set up a triple integral in spherical coordinates for the volume of the region bounded by $z = \sqrt{x^2 + y^2}$ and $z = 2$.

13.7 21. Set up a triple integral in spherical coordinates for $\iiint\limits_Q \sqrt{x^2+y^2}\, dV$ where Q is the region that lies between the spheres $x^2 + y^2 + z^2 = 4$ and $x^2 + y^2 + z^2 = 16$ and inside the cone $z = \sqrt{9x^2 + 9y^2}$.

13.7 22. Evaluate $\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{8-x^2-y^2}} (x^2 + y^2 + z^2)\, dz\, dy\, dx$

13.8 23. Find the surface area of the portion of the paraboloid $z = 1 + x^2 + y^2$ that lies above the unit disk $x^2 + y^2 \leq 1$.

13.9 24. Evaluate $\iint\limits_R y^2 e^{xy}\, dA$ where R is bounded by $xy = 1, xy = 2, y = 3$ and $y = 4$.

14.1 25. Let $\mathbf{F} = x^3y^2z\mathbf{i} + xyz\mathbf{j} + xy\mathbf{k}$. Find the divergence and curl of $\mathbf{F}$.

14.2 26. Evaluate $\int\limits_C (x + 2)\, ds$, where C is given by: $x = t, y = \frac{4}{3}t^{1.5}, z = \frac{t^2}{2}$ for $0 \leq t \leq 2$.

14.2 27. Find the work done by $\mathbf{F} = [x, 2y]$ on a particle moving along $y = x^3$ from $(0, 0)$ to $(2, 8)$.

28. Is $\mathbf{F}$ conservative? If so, find a potential function for $\mathbf{F}$.

14.3 a) $\mathbf{F} = [y \sin x, \cos x]$

b) $\mathbf{F} = [\frac{1}{y}, -\frac{x}{y^2}, 2z - 1]$

14.3 29. If possible, evaluate $\int\limits_C \mathbf{F} \cdot d\mathbf{r}$ where C begins at $(2,3)$ and ends at $(7,6)$.

a) $\mathbf{F} = [xy^2, x^2y]$
b) $\mathbf{F} = [y, -x]$

14.4 30. Evaluate $\int\limits_C [y^3dx + (x^3 + 3xy^2)dy]$ where C is the part of $y = x^2$ from $x = 0$ to $x = 2$ followed by the straight line segment from $(2,4)$ to $(0,0)$.

14.4 31. Let C be the upper semi-circle of radius 9 centred at the origin. Calculate the flux of $\mathbf{F} = [2x^3 + 3y, 2y^3 + 4x]$ across C.

14.5 32. Evaluate $\iint\limits_S z\, dS$ where S is given by: $x = 4u\cos v, y = 4u\sin v, z = 3u$ for $0 \le u \le 4$ and $0 \le v \le 2\pi$.

14.5 33. Compute the flux of $\mathbf{F} = [xz, 0, z^2]$ across the first octant of the unit sphere.

14.6 34. Find the flux of $\mathbf{F} = [xe^z, ye^z, e^z]$ across the surface bounded by $z = 4 - y, z = 0$, $y = 0, x = 0$ and $x = 6$.

OMIT ~~35. Evaluate $\oint\limits_C \mathbf{F} \cdot \mathbf{T}\, ds$ where $\mathbf{F} = [y^2, -z, -x]$ and C is the boundary of $2x + 2y + z = 6$ in the first octant, oriented counter-clockwise when viewed from above.~~