Path Independence Theorem

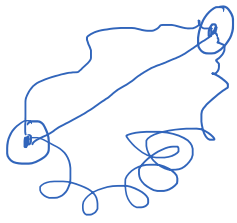
$$\int_C \vec{F} \cdot d\vec{r} = f(B) - f(A)$$

(B, A: endpoints of C)

if and only if $\vec{F} = \nabla f$

Means:

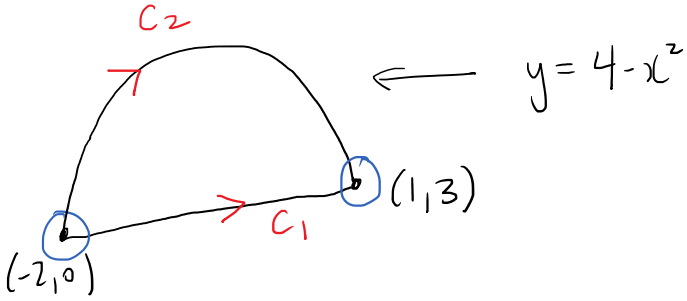
1) If $\vec{F} = \nabla f$ then the value of $\int_C \vec{F} \cdot d\vec{r}$ does not depend on the path C (the integral is "path independent")



why? $\int_C \nabla f \cdot d\vec{r} = f(B) - f(A)$

2) If $\vec{F} \neq \nabla f$ then the path matters.
Different paths C give different values for the integral.
(The integral is "path dependent").

Ex:



Does $\int_{C_1} (y^2 dx + 2xy dy) = \int_{C_2} (y^2 dx + 2xy dy)$?

Integrand $= \vec{F} \cdot [dx, dy]$

$\vec{F} = [y^2, 2xy]$

$\frac{\partial P}{\partial y} = 2y = \frac{\partial Q}{\partial x}$

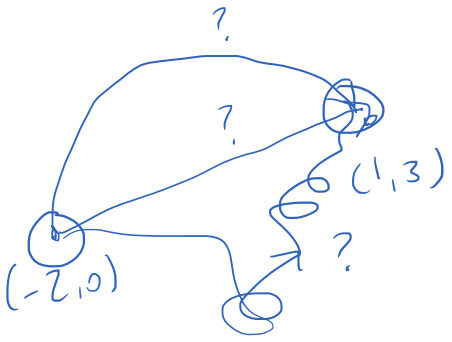
$\vec{F} = \nabla f$

YES

(Aside: Integral $= xy^2 \Big|_{(-2,0)}^{(1,3)} = 9$)

Ex: Let C be a path from $(-2,0)$ to $(1,3)$

Is $\int_C (xy\,dx + 2\,dy)$ path dependent?



$\vec{F} \cdot [dx, dy]$

$\vec{F} = [xy, 2]$

$\frac{\partial P}{\partial y} = x \qquad \frac{\partial Q}{\partial x} = 0$

$\frac{\partial P}{\partial y} \neq \frac{\partial Q}{\partial x}$

$\vec{F} \neq \nabla f$

Yes

Need to know the path in order to evaluate the integral.