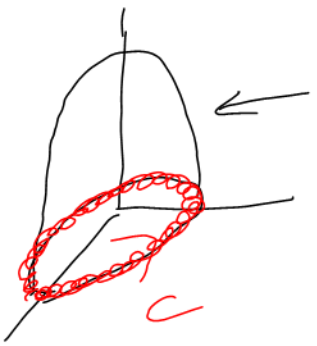
(1)



Surface S

$$z = \sqrt{4 - x^2 - y^2}$$

C: $z = z$

$$0 = \sqrt{4 - x^2 - y^2}$$

$$0 = 4 - x^2 - y^2$$

$x^2 + y^2 = 4$ (circle of radius 2)

Cylindrical $x = r\cos\theta \quad y = r\sin\theta \quad z = 0$

$x = 2\cos t \quad y = 2\sin t \quad z = 0$

$dx = -2\sin t\,dt \quad dy = 2\cos t\,dt \quad dz = 0$

$0 \le t \le 2\pi$

$$\vec{F} = [3y, -2x, xyz]$$

$$\iint_S (\text{curl}\,\vec{F}) \cdot \vec{n}\, dS = \oint_C \vec{F} \cdot \vec{T}\, ds$$

$$= \oint_C (P\,dx + Q\,dy + R\,dz)$$

→

(1) Cont'd

$$= \oint_C (3y\,dx - 2x\,dy + xyz\,dz)$$

$$= \int_0^{2\pi} [-12\sin^2 t - 8\cos^2 t]\,dt$$

Recall $\sin^2 t = \frac{1-\cos 2t}{2}$

$\cos^2 t = \frac{1+\cos 2t}{2}$

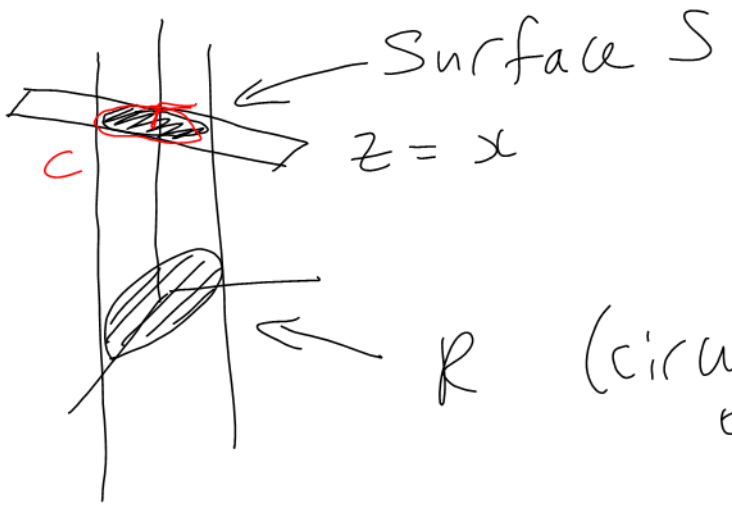
$$= \int_0^{2\pi} [-6 + 6\cos 2t - 4 - 4\cos 2t]\,dt$$

$$= \int_0^{2\pi} [-10 + 2\cos 2t]\,dt$$

$$= [-10t + \sin 2t]_0^{2\pi}$$

$$= -20\pi$$

(7)



$$\vec{F} = [2z, x, 3y]$$

$$\text{curl}\,\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2z & x & 3y \end{vmatrix}$$

$$= \vec{i}(3) - \vec{j}(-2) + \vec{k}(1)$$

$$= [3, 2, 1]$$

$$z = x$$

$$\Rightarrow z_x = 1, \quad z_y = 0$$

$$\vec{n}\,dS = \pm[-z_x, -z_y, 1]\,dy\,dx$$

$$= \pm[-1, 0, 1]\,dy\,dx$$

Choose $\oplus$ for upwards

$\rightarrow$

⑦ Cont'd

$$\oint_C \vec{F} \cdot \vec{T}\, ds = \iint_S (\text{curl}\, \vec{F}) \cdot \vec{n}\, dS$$

$$= \iint_R [3,2,1] \cdot [-1,0,1]\, dy\, dx$$

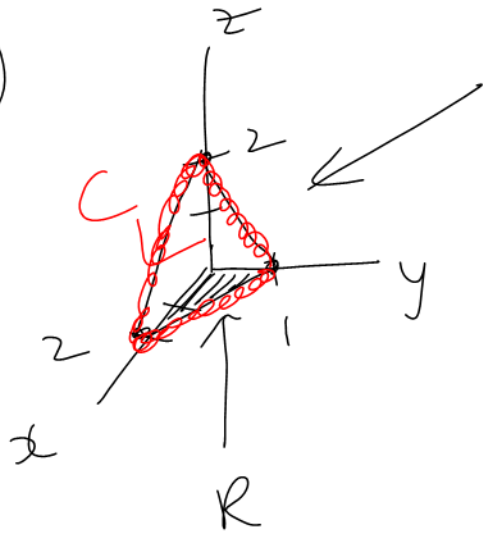
$$= \iint_R -2\, dy\, dx$$

$$= -2 \boxed{\iint_R dy\, dx} \leftarrow \text{area of } R$$

$$= -2(4\pi)$$

$$= -8\pi$$

(9)



Surface S

$x+2y+z=2$

$z=2-x-2y$

$$\vec{F}=[y-x,\ x-z,\ x-y]$$

$$curl\,\vec{F}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ y-x & x-z & x-y\end{vmatrix}$$

$$=\vec{i}(-1+1)\ -\vec{j}(1)+\vec{k}(1-1)$$

$$=[0,-1,0]$$

Surface $z=2-x-2y$

$z_x=-1,\quad z_y=-2$

$$\vec{n}\,dS=\pm[-z_x,-z_y,1]\,dy\,dx$$

$$=\pm[1,2,1]\,dy\,dx$$

Choose $\oplus$ for upwards

→

R:

$$\oint_C \vec{F} \cdot \vec{T}\, ds = \iint_S (\text{curl}\,\vec{F}) \cdot \vec{n}\, dS$$

$$= \iint_R [0,-1,0] \cdot [1,2,1]\, dy\, dx$$

$$= \iint_R -2\, dy\, dx$$

$$= -2 \boxed{\iint_R dy\, dx} \leftarrow \text{area of } R$$

$$= -2\,(1)$$

$$= -2$$