

1. [3 marks] Evaluate:

$$\lim_{x \rightarrow -4} \frac{x^2 - 3x - 28}{2x^2 - 32}$$

$$= \lim_{x \rightarrow -4} \frac{(x-7)(x+4)}{2(x^2-16)}$$

$$= \lim_{x \rightarrow -4} \frac{(x-7)(x+4)}{2(x+4)(x-4)}$$

$$= \lim_{x \rightarrow -4} \frac{x-7}{2(x-4)}$$

$$= \frac{-11}{-16} \text{ or } \frac{11}{16}$$

2. [6 marks] Find y' :

a) $y = \arctan x^2 + \arcsin 3x$

$$y' = \frac{1}{1+(x^2)^2} (2x) + \frac{1}{\sqrt{1-(3x)^2}} (3)$$
$$= \frac{2x}{1+x^4} + \frac{3}{\sqrt{1-9x^2}}$$

b) $y = 3e^{4x} + 2\ln(x^3 + 1)$

$$y' = 12e^{4x} + \frac{2}{x^3+1} (3x^2)$$
$$= 12e^{4x} + \frac{6x^2}{x^3+1}$$

c) $y = \sec 2x \cos^3 x$

$$y = \sec 2x [6 \cos^2 x]^3$$
$$y' = \sec 2x [3 \cos^2 x (-\sin x)] + 6 \cos^3 x [\sec 2x \tan 2x (2)]$$
$$= -3 \cos^2 x \sin x \sec 2x + 2 \cos^3 x \sec 2x \tan 2x$$

3. [4 marks] Find $\frac{dy}{dx}$ given:

$$9xy^2 - 4x^2y + 3x^4 = 12x$$

$$9x(2y \frac{dy}{dx}) + 9y^2 - 4x^2 \frac{dy}{dx} - 8xy + 12x^3 = 12$$

$$18xy \frac{dy}{dx} - 4x^2 \frac{dy}{dx} = 12 - 9y^2 + 8xy - 12x^3$$

$$[18xy - 4x^2] \frac{dy}{dx} = 12 - 9y^2 + 8xy - 12x^3$$

$$\frac{dy}{dx} = \frac{12 - 9y^2 + 8xy - 12x^3}{18xy - 4x^2}$$

$$\text{or } \frac{-12 + 9y^2 - 8xy + 12x^3}{-18xy + 4x^2}$$

16. [3 marks] Find $\int \frac{dx}{x^2+10x+29}$.

Complete the square:

$$\begin{aligned}x^2+10x+29 &= (x+5)^2+4 \\ &= (x+5)^2+2^2\end{aligned}$$

$$\text{Integral} = \int \frac{dx}{(x+5)^2+2^2}$$

$$\begin{array}{l}u = x+5 \\ du = dx\end{array}$$

$$= \int \frac{du}{u^2+2^2}$$

$$= \frac{1}{2} \tan^{-1} \frac{u}{2} + C$$

$$= \frac{1}{2} \tan^{-1} \frac{x+5}{2} + C$$

5. [3 marks] Find $\int \frac{1+e^{2x}}{2x+e^{2x}} dx$.

$$\begin{aligned} u &= 2x + e^{2x} \\ du &= (2 + 2e^{2x}) dx \\ \frac{du}{2} &= (1 + e^{2x}) dx \end{aligned}$$

$$= \frac{1}{2} \int \frac{du}{u}$$

$$= \frac{1}{2} \ln|u| + C$$

$$= \frac{1}{2} \ln|2x + e^{2x}| + C$$

4. [3 marks] Evaluate $\int_2^4 \frac{7x^2}{(x^3-1)^2} dx$.

$$\begin{aligned} u &= x^3 - 1 \\ du &= 3x^2 dx \\ \frac{du}{3} &= x^2 dx \\ x=2 &\Rightarrow u=7 \\ x=4 &\Rightarrow u=63 \end{aligned}$$

$$\begin{aligned} &= \frac{7}{3} \int_7^{63} \frac{du}{u^2} \\ &= \frac{7}{3} \int_7^{63} u^{-2} du \\ &= \frac{7}{3} \left[-u^{-1} \right]_7^{63} \\ &= \frac{7}{3} \left[\frac{-1}{63} + \frac{1}{7} \right] \\ &= \frac{7}{3} \left(\frac{8}{63} \right) \\ &= \frac{8}{27} \end{aligned}$$

7. [3 marks] Find $\int \frac{\sin \sqrt{x}}{\sqrt{x} \cos \sqrt{x}} dx$.

$$= \int \frac{\tan \sqrt{x}}{\sqrt{x}} dx$$

$$\begin{aligned} u &= \sqrt{x} \\ du &= \frac{1}{2} x^{-1/2} dx \\ 2 du &= \frac{dx}{\sqrt{x}} \end{aligned}$$

$$= 2 \int \tan u du$$

$$= 2 \ln |\sec u| + C$$

$$= 2 \ln |\sec \sqrt{x}| + C$$

$$\text{OR } -2 \ln |\cos \sqrt{x}| + C$$