

1. [3 marks] Evaluate:

$$\lim_{x \rightarrow -5} \frac{3x^2 - 75}{7x^2 + 35x}$$

$$= \lim_{x \rightarrow -5} \frac{3(x^2 - 25)}{7x(x+5)}$$

$$= \lim_{x \rightarrow -5} \frac{3(x+5)(x-5)}{7x(x+5)}$$

$$= \lim_{x \rightarrow -5} \frac{3(x-5)}{7x}$$

$$= \frac{-30}{-35} \quad \text{or} \quad \frac{6}{7}$$

2. [6 marks] Find y' :

a) $y = \arctan x^3 + \arcsin 2x$

$$y' = \frac{1}{1+(x^3)^2} (3x^2) + \frac{1}{\sqrt{1-(2x)^2}} (2)$$
$$= \frac{3x^2}{1+x^6} + \frac{2}{\sqrt{1-4x^2}}$$

b) $y = 3e^{7x} + 2\ln(x^2 + 1)$

$$y' = 21e^{7x} + \frac{2}{x^2+1} (2x)$$
$$= 21e^{7x} + \frac{4x}{x^2+1}$$

c) $y = \sec 2x \cos^4 x$

$$y = \sec 2x [\cos x]^4$$

$$y' = \sec 2x [4(\cos x)^3 (-\sin x)] + \cos^4 x [\sec 2x \tan 2x (2)]$$
$$= -4 \cos^3 x \sin x \sec 2x + 2 \cos^4 x \sec 2x \tan 2x$$

3. [4 marks] Find $\frac{dy}{dx}$ given:

$$7x^3 - 6x^2y + 7xy^2 = 13x$$

$$21x^2 - 6x^2 \frac{dy}{dx} + y(-12x) + 7x(2y \frac{dy}{dx}) + 7y^2 = 13$$

$$-6x^2 \frac{dy}{dx} + 14xy \frac{dy}{dx} = 13 - 21x^2 - 7y^2 + 12xy$$

$$[-6x^2 + 14xy] \frac{dy}{dx} = 13 - 21x^2 - 7y^2 + 12xy$$

$$\frac{dy}{dx} = \frac{13 - 21x^2 - 7y^2 + 12xy}{-6x^2 + 14xy}$$

$$\text{or } \frac{-13 + 21x^2 + 7y^2 - 12xy}{6x^2 - 14xy}$$

16. [3 marks] Find $\int \frac{dx}{x^2+8x+25}$.

Complete the Square:

$$\begin{aligned}x^2+8x+25 &= (x+4)^2+9 \\ &= (x+4)^2+3^2\end{aligned}$$

$$\text{Integral} = \int \frac{dx}{(x+4)^2+3^2}$$

$$\boxed{\begin{aligned}u &= x+4 \\ du &= dx\end{aligned}}$$

$$= \int \frac{du}{u^2+3^2}$$

$$= \frac{1}{3} \tan^{-1} \frac{u}{3} + C$$

$$= \frac{1}{3} \tan^{-1} \frac{x+4}{3} + C$$

5. [3 marks] Find $\int \frac{1+e^{3x}}{3x+e^{3x}} dx$.

$$= \frac{1}{3} \int \frac{du}{u}$$

$$= \frac{1}{3} \ln|u| + C$$

$$= \frac{1}{3} \ln|3x+e^{3x}| + C$$

$$\begin{aligned} u &= 3x + e^{3x} \\ du &= (3 + 3e^{3x}) dx \\ \frac{du}{3} &= (1 + e^{3x}) dx \end{aligned}$$

4. [3 marks] Evaluate $\int_2^3 \frac{6x}{(x^2-1)^2} dx$.

$$\begin{aligned} u &= x^2 - 1 \\ du &= 2x dx \\ \frac{du}{2} &= x dx \\ x=2 &\Rightarrow u=3 \\ x=3 &\Rightarrow u=8 \end{aligned}$$

$$= 3 \int_3^8 \frac{du}{u^2}$$

$$= 3 \int_3^8 u^{-2} du$$

$$= 3 [-u^{-1}]_3^8$$

$$= 3 \left[-\frac{1}{8} + \frac{1}{3} \right]$$

$$= 3 \left(\frac{5}{24} \right)$$

$$= \frac{5}{8}$$

7. [3 marks] Find $\int \frac{\sin(\ln x)}{x \cos(\ln x)} dx$.

$$= \int \frac{\tan(\ln x)}{x} dx$$

$$\begin{array}{l} u = \ln x \\ du = \frac{1}{x} dx \end{array}$$

$$= \int \tan u du$$

$$= \ln |\sec u| + C$$

$$= \ln |\sec(\ln x)| + C$$