SECTION 1. Collection and Representation of Data

Ex 1. Loudness of jet engines at takeoff (decibels):
102, 115, 93, 105, 108, 110, 120, 94, 101, 103,
92, 110, 109, 101, 115, 119, 95, 108, 98, 114
a) Create a frequency table with six classes
b) Draw a histogram

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SECTION 2. Summarizing Data

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Ex 1. Find the mean of the sample $11, 9, 17, 19, 4, 15$.

Ex 2. A student has test marks 58, 63 and 71. What mark on his 4th test gives him an average of 70?

Ex 3. If the two populations are combined into one, find the new mean.

	Population Size	μ
Pop. 1	43	71
Pop. 2	26	68

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Ex 4. Find the sample mean:

Temperature (°C)	Frequency
22	11
23	6
25	3

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Ex 5. Find the sample mean:

mass (g)	relative frequency
82	0.55
86	0.4
88	0.05

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Ex 6. Find the median:
a) $2, 9, 11, 5, 6$
b) $2, 9, 11, 5, 6, 10$

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Ex 7. Find the median for the data in Example 4

Ex 8. Find the median for the data in Example 5

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Ex 9. Find the population SD of $2, 5, 8, 9$

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Ex 10. Find the sample SD of $12, 13, 17$

Ex 11. Which sample is more spread out?
a) $1, 4, 10$
b) $31, 36, 38$

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Ex 12. Two machines are filling 355 mL cans of pop. A sample of volumes has the following means and variances (in mL).

	Machine 1	Machine 2
$\overline{x}$	355.8	355.2
s^2	0.3	1.4

a) Which machine is more accurate?
b) Which machine is more precise?

Ex 13. Let a population consist of the salaries at a small engineering firm. What happens to the mean, median and SD in each situation:
a) Each employee get a $2,000 raise?
b) Each employee's salary is doubled?

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SECTION 3. Probability

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Ex 1. Flip a fair coin three times. What is the probability of getting one or two heads?

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Ex 2. Randomly select an integer between 1 and 40 (inclusive). Find the probability of getting a multiple of 5 or 7.

Ex 3. Roll a pair of fair 6-sided dice. Find the probability of getting a sum of at most 5.

Ex 4. Four study groups have the following numbers of students: 4, 6, 7, 9. Pick two of the groups at random. Find the probability that they have at least 15 students in total.

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Ex 5. Consider the following data on students' food preferences:

	Likes Coffee	Dislikes Coffee
Likes Spicy Food	19	1
Dislikes Spicy Food	3	17

Find the probability that a student:
a) likes coffee
b) likes coffee and dislikes spicy food
c) likes coffee or dislikes spicy food

Ex 6. In a class of 45 students, 26 have jobs and 17 have cars. Of those who don't have a car, 10 have jobs. Find the probability that a randomly chosen student has:
a) a car or a job
b) a car but not a job

Ex 7. On any given day, the probability that Machine I breaks down is 4%, the probability that Machine II breaks down is 7%, and the probability that both machines break down is 2%. Find the probability that Machine II breaks down and Machine I doesn't.

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Ex 8. A password consists of 7 digits, each chosen from $0,1,2,\cdots,9$. Find:
a) the total number of possible passwords
b) the number of passwords that end with 3
c) the number of passwords that don't end with 3
d) the probability that a password starts with 4

e) the probability that a password doesn't start with 4
f) the probability that a password contains a least one 4
g) the probability that a password starts with 29 or ends with 1

SECTION 4. Discrete Random Variables

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Ex 1. Let X be the number of heads observed in 3 coin tosses. Find the probability distribution of X.

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Ex 2. Given the following probability distribution:

x	$P(x)$
-5	0.15
-2	0.2
1	0.4
6	0.25

Find:
a) $P(-2.5 \leq X \leq 2.5)$
b) the mean of X
c) the variance of X
d) the standard deviation of X
e) the probability that an x-value lies within one standard deviation of the mean

Ex 3. Project 1 has a 35% chance of earning \$0, a 50% chance of earning \$300,000 and a 15% chance of earning \$800,000.
Project 2 has a 60% chance of earning \$0 and a 40% chance of earning \$1,000,000.

a) Find the probability distributions of the earnings for each project
b) Find the expected earnings for each project
c) Find the standard deviation of earnings for each project
d) Which project has higher expected earnings?
e) In terms of earnings, which project is riskier?

Ex 4. Suppose you want to insure a \$2,000 tablet against theft for one year by paying a premium m. The probability of theft is 4.7%.
a) Find the probability distribution of the insurance company's gain
b) Find the premium if the insurance company expects to gain \$40

SECTION 5. Binomial and Poisson Distributions

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Ex 1. Roll a die 13 times. Find the probability of getting at most three 2's or 3's.

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Ex 2. A drilling company is successful on 82% of drilling attempts. Find the probability of at least seven successes in the next eight attempts.

Ex 3. A dart-thrower hits the target 36% of the time. He does not improve with practice. He throws ten darts. Find the probability that he hits the target two or three times.

Ex 4. A multiple-choice test has three questions, each of which has four possible answers. A student guesses randomly on each question.
a) Find the probability distribution for the number of questions the student gets correct
b) Draw a histogram

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Ex 5. In a city, the average number of cracks per cubic meter of concrete is 1.9. Find the probability that a randomly-chosen cubic meter of concrete has:
a) 2 or 3 cracks
b) at least 3 cracks

Ex 6. A website receives an average of 2 visits per hour.
a) Find the probability distribution for the number of visits in the next hour
b) Draw a histogram

Ex 7. Suppose that the concentration of bacteria in the Inner Harbour is 3 bacteria per 100 mL of water. Find the probability that there are at most 2 bacteria in a 50 mL sample of water.

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Ex 8. There are an average of 1.8 accidents per week on a certain highway. Find the probability that there will be at least 4 accidents on that highway in the next 2 weeks.

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SECTION 6. Continuous Random Variables

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Ex 1. The p.d.f. for a continuous random variable X is

$$f(x) = \begin{cases} \frac{x}{8} & \text{if } 0 < x \leq 2 \\ \frac{1}{4} & \text{if } 2 < x \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

Find:
a) $P(X = 2.2)$
b) $P(1 \leq X \leq 3)$
c) $P(1 < X < 3)$
d) $P(X > 1.2)$
e) $P(X < 0.6)$

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Ex 2. Find the value of k that makes $f(x)$ a valid p.d.f.

$$f(x) = \begin{cases} kx^7 & 1 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Ex 3. In a certain city, the daily consumption of electric power (in millions of kilowatt hours) has p.d.f.

$$f(x) = \begin{cases} \frac{x}{9}e^{-x/3} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

If the city's daily supply is 12 million kilowatt hours, what is the probability that the supply will be inadequate on any given day?

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Ex 4. The proportion X of a city's roads needing repair in any given year has p.d.f.

$$f(x) = \begin{cases} 12x^2 - 12x^3 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

a) Graph $f(x)$ using Wolfram Alpha
b) Find the expected proportion of roads needing repairs this year
c) Find the SD of X

Ex 5. Find the average daily consumption of electric power using Example 3.

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Ex 6. Consider a train wheel of radius r. Let x be the location along the circumference (relative to some fixed point 0) at which the wheel makes contact with the rail upon braking. Then X has p.d.f.

$$f(x) = \begin{cases} \dfrac{1}{2\pi r} & 0 \leq x < 2\pi r \\ 0 & \text{otherwise} \end{cases}$$

a) Find the mean of X
b) Find the variance of X

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Ex 7. The mileage (in thousands of miles) for a certain kind of tire has p.d.f.

$$f(x) = \begin{cases} \frac{1}{20}e^{-x/20} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

Find the probability that a tire will last:
a) exactly 10,000 miles
b) at most 10,000 miles
c) at least 30,000 miles
d) between 16,000 and 24,000 miles

Ex 8. If the number of arrivals per unit time is λ, then the waiting time between arrivals is an exponential random variable with $k = \lambda$. Suppose that, on average, 3 trucks arrive at a warehouse per hour. The time (in hours) between truck arrivals has p.d.f.

$$f(x) = \begin{cases} 3e^{-3x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

Find the probability that the time between arrivals is:
a) less than five minutes
b) more than 45 minutes

SECTION 7. The Normal Distribution

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Ex 1. The volume in bottles of ginger ale is normally distributed with a mean of 2.01 L and a SD of 0.13 L. Find the probability that a bottle has a volume:
a) between 1.77 and 2.29 L
b) between 1.59 and 1.73 L
c) less than 1.81 L

Ex 2. Find the proportion of x-values that are within one standard deviation of the mean for a normal distribution.

Ex 3. The mass of Choco chocolate bars is normal with a mean of 85g and a SD of 1.5g. Find the mass that separates the largest 32% of chocolate bar masses from the others.

Ex 4. The lengths of drill bits are normal with a mean of 4.2cm and a SD of 1.1cm. Find the length that separates the smallest 15% of drill bit lengths from the others.

Ex 5. The time it takes to inspect a certain type of production component is normally distributed with a mean of 6.8 s. Find the SD of the inspection times if 26.62% of inspection times are between 6.2 and 7.4 s.

SECTION 8. The Central Limit Theorem

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Ex 1. A large class has a test average of 72 with a SD of 8. Take a random sample of n tests. Find the probability that the average of the n tests is more than 75 if:
a) $n = 30$
b) $n = 80$

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Ex 2. Checked baggage has a mean mass of 21 kg with a SD of 4 kg. If 40 bags are randomly selected, find the probability that their average mass is:
a) between 20 and 23 kg
b) less than 20 kg or more than 22kg

Ex 3. Checked baggage has a mean mass of 21 kg with a SD of 4 kg. Find the probability that the total mass of 50 random bags is greater than 1130 kg.

Ex 4. Ball bearings have an average radius of 9.9mm with a SD of 1.4mm. Take a random sample of 60 ball bearings. Find c so that $P(\overline{x} > c) = 0.97$

Ex 5. Ball bearings have an average radius of 9.9mm with a SD of 1.4mm. How large a sample would be required to ensure that $P(\overline{x} \geq 9.6) \geq 0.99$?

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SECTION 9. Confidence Intervals

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Ex 1. Volumes in cans of Coke have a SD of 2.5 mL. A random sample of 60 cans had an average volume of 355.3 mL. Find a 95% confidence interval for the average volume among all cans of Coke.

Ex 2. Consider a 99% confidence interval with the same σ, n and $\overline{x}$ as the previous example. Would it be wider or narrower than the 95% confidence interval in the previous example?

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Ex 3. Fifty random Camosun students were polled on their number of googles last week. The average was 23.4 with a SD of 3.7. Find a 90% confidence interval for the average number of googles last week among all Camosun students.

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Ex 4. A sample of lightbulb lifetimes had a SD of 8.9 months. We want to estimate the population mean with a 90% margin of error less than 0.1 months. What is the minimum sample size we should use?

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Ex 5. Thirty randomly-selected water samples have a mean pollution concentration of 48.1 ppm with a standard deviation of 6.2 ppm. Find a 99% UCB for the mean pollution concentration in the body of water.

Ex 6. In a large class, test marks have a SD of 10.3. A random sample of 40 tests has an average mark of 69.1. Find a 98% LCB for the class average.

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Ex 7. The radius of ball bearings is normally distributed. A random sample of 10 ball bearings has a mean radius of 4.9 cm with a SD of 0.9 cm. Find a 95% confidence interval for the mean radius.

Ex 8. The masses of Choco chocolate bars are normal. A random sample of 18 bars had a mean mass of 84.7 g and a SD of 2.6 g. Find a 99% UCB for the mean mass among all Choco bars.

Ex 9. The breaking strengths of a certain brand of rope are normal. A random sample of 8 ropes had a mean breaking strength of 62.1 lbs with a SD of 2.5 lbs. Find a 90% LCB for the mean breaking strength.

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SECTION 10. Linear Regression

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Ex 1. Data Set A has $r = 0.73$ and Data Set B has $r = -0.85$. Which data set has a stronger linear association?

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Ex 2. A bivariate data set has $r = -0.8$. What percentage of the variation in y is accounted for by the best-fit line?

Ex 3. Given: $\hat{y} = 5.61 - 0.13x$ and the coefficient of determination is 0.9522.

x=age of Toyota Corolla (years)	y=resale value (\$1000s)
2	5.4
3	5.1
5	4.9
7	4.8
10	4.2

a) Is the linear association positive or negative?
b) Find the correlation coefficient.
c) What % of the variation in y is accounted for by the best-fit line?
d) What resale value is predicted for a 4-year-old Corolla?
e) Why should we not predict the resale value for a 1-year-old Corolla?
f) What age corresponds to a resale value of $4,500?
g) Interpret the slope of $\hat{y}$

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Ex 4. Find $\hat{y}$ and r^2

x	y
2	5
8	4
9	2

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