1. ~~[4~~ 2 marks] On a certain road, there were 80 accidents over a 250-day period. Find the probability that there is more than one accident on that road tomorrow.

$X =$ # accidents tomorrow

Poisson $\frac{80}{250} = 0.32$ accidents/day

$\boxed{\text{Use } \mu = 0.32}$ 0.5

$P(X>1) = 1 - P(X=0) - P(X=1)$ 1

$= 1 - e^{-0.32}\left(\frac{0.32^0}{0!} + \frac{0.32^1}{1!}\right)$ 0.5

≈ 0.04

2. ~~[4~~ 2 marks] A skilled marksman hits a target 91% of the time. He does not improve with practice. What is the probability that he hits the target at most 17 times on 20 attempts?

$X =$ # successful hits

Binomial $n=20$ $p=0.91$ $q=1-p=0.09$ 0.5

$P(X \le 17) = 1 - P(X=18) - P(X=19) - P(X=20)$ 1

$= 1 - {}_{20}C_{18}(0.91)^{18}(0.09)^2$

$- {}_{20}C_{19}(0.91)^{19}(0.09)^1$

$- {}_{20}C_{20}(0.91)^{20}(0.09)^0$

≈ 0.27 0.5

3. [~~3~~ 2 marks] An engineering firm has 25 employees: 17 are full-time and the rest are part-time. Four employees are randomly selected. Find the probability that two or three of them are full-time employees.

17 FT/8 PT

$n(S) = 25C4 = 12650$

$n(2 \text{ or } 3\ FT) = n(2\ FT) + n(3\ FT)$

$= n(2\ FT \text{ and } 2\ PT)$

$+ n(3\ FT \text{ and } 1\ PT)$

$= 17C2 \times 8C2 + 17C3 \times 8C1$

$= 9248$

$P(2 \text{ or } 3\ FT) = \frac{9248}{12650} \approx 0.73$

1

1

4. [3 marks] A ticket for a charity lottery costs \$25. You have a 2.7% chance of winning \$700 and a 97.3% chance of winning nothing. Let your net gain=amount won−ticket cost.

a) Find the probability distribution of your net gain.

Net Gain (\$)	Probability
$700-25=675$	0.027
$0-25=-25$	0.973

2

b) Find your expected net gain.

$E(X) = 675(0.027) - 25(0.973)$

$= -6.1$

Expect to lose \$6.10.

1

5. [5 marks] The lifetime of a machine part (in years) has probability density function $f(x) = \begin{cases} 7e^{-7x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$

Find the probability that a part lasts:

a) exactly 0.3 years

$P(X=0.3) = 0$

X is a continuous variable

1

b) between 0.3 and 0.5 years

$$P(0.3<X<0.5) = \int_{0.3}^{0.5} 7e^{-7x}\,dx$$

$$= \left[-e^{-7x}\right]_{0.3}^{0.5}$$

$$= -e^{-3.5} + e^{-2.1}$$

$$\approx 0.09$$

2

c) at most 0.3 years

$$P(X<0.3) = \int_0^{0.3} 7e^{-7x}\,dx + \cancel{\int_{-\infty}^{0} 0\,dx}$$

$$= \left[-e^{-7x}\right]_0^{0.3}$$

$$= -e^{-2.1} + 1$$

$$\approx 0.88$$

2

-1 for integration error -0.5 for evaluation error

6. [3 marks] Employees at a large engineering firm worked a mean of 51 hours last week, with a standard deviation of 3 hours. Four hundred employees are selected at random. Find the probability that their work last week averaged less than 50.5 hours.

$\mu = 51 \quad \sigma = 3 \quad n = 400 \quad \bar{x} =$ sample mean

$P(\bar{x} < 50.5)$

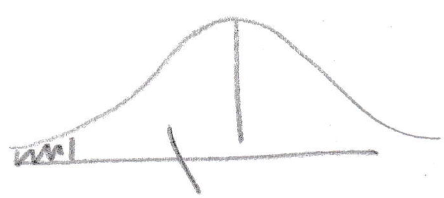
$$\left\{ \begin{aligned} z &= \frac{\bar{x} - \mu}{(\sigma / \sqrt{n})} \\ z &= \frac{50.5 - 51}{(3/\sqrt{400})} \\ &\approx -3.33 \end{aligned} \right.$$

1

1

$= P(z < -3.33)$

$=$



0.4996

$= 0.5 - 0.4996$

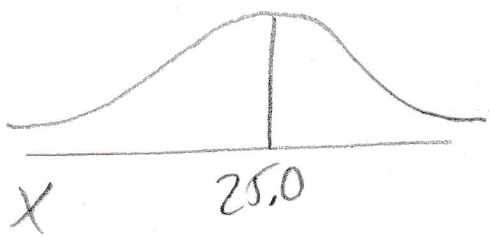
$= 0.0004$

1

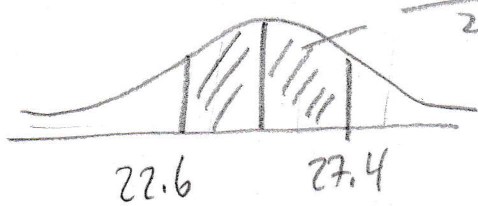
7. [3 marks] The masses of ball bearings at a manufacturing plant are normally distributed with a mean of 25.0g. Find the standard deviation of the masses if 45.14% of the ball bearings have masses between 22.6g and 27.4g.

X = mass (g)

X is normal $\qquad \mu = 25.0\text{g}$



$\frac{0.4514}{2} = 0.2257$



Reverse look-up area = 0.2257 $\quad$ 1

$z = 0.6$

$\boxed{z = 0.6 \text{ corresponds to } X = 27.4}$ $\quad$ 1

$z = 0.6,\ X = 27.4,\ \mu = 25.0$

$\rightarrow \quad z = \frac{X - \mu}{\sigma}$

$0.6 = \frac{27.4 - 25.0}{\sigma}$

$0.6 = \frac{2.4}{\sigma}$

$\sigma = \frac{2.4}{0.6} \approx 4\text{g}$ $\quad$ 1