1. [2 marks] Solve $y'' - y' - 30y = 0$

$$m^2 - m - 30 = 0$$

$$(m-6)(m+5) = 0$$

$$m = 6, -5$$

$$y = C_1 e^{6x} + C_2 e^{-5x}$$

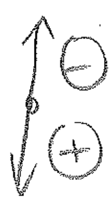
2. [3 marks] Set up the DE and the initial conditions.
DO NOT SOLVE THE DE.

A 7 kg mass is attached to a spring with spring constant 2.3 N/m. There is a damping force equal to 2 times the velocity, as well as an external force $f(t) = 4\cos 2t$. The mass is initially 15 cm above the equilibrium position with an upwards velocity of 10 cm/s.

$$mx'' = -\beta x' - kx + f(t)$$

$$\boxed{7x'' = -2x' - 2.3x + 4\cos 2t}$$

MARKS: (0.5 each term)



$$\boxed{\begin{aligned} x(0) &= -0.15 \text{ m} \\ x'(0) &= -0.1 \text{ m/s} \end{aligned}}$$

-0.5 for signs/units

MARKS:

3. [4 marks] Solve $\frac{d^4y}{dx^4} - 4\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} = 0$

$$m^4 - 4m^3 + 6m^2 = 0$$

$$m^2(m^2 - 4m + 6) = 0$$

$$m = 0, 0$$

$$m = \frac{4 \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 6}}{2}$$

$$m = \frac{4 \pm \sqrt{-8}}{2} \quad \sqrt{4}\sqrt{2}\sqrt{-1}$$

$$m = \frac{4 \pm 2\sqrt{2}\,j}{2}$$

$$m = 2 \pm \sqrt{2}\,j$$

$$(\alpha = 2,\ \beta = \sqrt{2})$$

$$y = e^{2x}(C_1 \sin\sqrt{2}x + C_2 \cos\sqrt{2}x) + C_3 e^{0x} + C_4 x e^{0x}$$

or $y = e^{2x}(C_1 \sin\sqrt{2}x + C_2 \cos\sqrt{2}x) + C_3 + C_4 x$

4. [6 marks] Solve $y'' - 5y' - 24y = 2e^{7x} - 9x$. (Write on the back of the page if you need extra space.)

1) y_c

$y'' - 5y' - 24y = 0$

$m^2 - 5m - 24 = 0$

$(m-8)(m+3) = 0$

$m = 8, -3$

$y_c = C_1 e^{8x} + C_2 e^{-3x}$

2) y_p

$y_p = Ae^{7x} + Bx + C$ not bad case

3) $y_p \to DE$

$$\begin{cases} y_p = Ae^{7x} + Bx + C \\ y_p' = 7Ae^{7x} + B \\ y_p'' = 49Ae^{7x} \end{cases}$$

$y'' - 5y' - 24y = 2e^{7x} - 9x$

$(49Ae^{7x}) - 5(7Ae^{7x} + B) - 24(Ae^{7x} + Bx + C)$
$= 2e^{7x} - 9x$

$49Ae^{7x} - 35Ae^{7x} - 5B - 24Ae^{7x} - 24Bx - 24C$
$= 2e^{7x} - 9x$ $\rightarrow$

$$-10Ae^{7x} - 24Bx - 5B - 24C = 2e^{7x} - 9x$$

or

$$-10Ae^{7x} - 24Bx + (-5B - 24C) = 2e^{7x} - 9x + 0$$

Match coefficients:

① $-10A = 2$

② $-24B = -9$

③ $-5B - 24C = 0$

①: $A = \frac{-1}{5}$

②: $B = \frac{9}{24}$

$B = \frac{9}{24} \to$ ③: $-5\left(\frac{9}{24}\right) - 24C = 0$

$$-24C = \frac{45}{24}$$

$$C = \frac{-45}{576}$$

$$\boxed{y_p = \frac{-1}{5}e^{7x} + \frac{9}{24}x - \frac{45}{576}}$$

4) $y = y_c + y_p$

$$y = C_1e^{8x} + C_2e^{-3x} - \frac{e^{7x}}{5} + \frac{9x}{24} - \frac{45}{576}$$

5. [5 marks] Consider the following population of test marks:

Mark	Frequency
65	8
70	12
75	10
80	11
85	9
90	3

a) Compute the mean

\# of measurements $n = 8 + 12 + \dots + 3 = 53$

$$\mu = \frac{65(8) + 70(12) + \dots + 90(3)}{53}$$

≈ 75.9

b) Compute the median

Position of median (when data is ordered) is $\frac{n+1}{2} = 27$

Median $= 27^{th}$ measurement

$= 75$

c) The population SD is 7.4. If each mark is decreased by 5, find the new population variance.

New pop. SD $= 7.4$ (spread is unchanged)

New pop variance $= (7.4)^2$

$= 54.76$

6. [5 marks] A PIN consists of 4 symbols, where each symbol can be 0 to 9. Find:

a) the total number of possible PIN's

$n(S) = 10 \times 10 \times 10 \times 10 = 10{,}000$

b) the probability that a PIN doesn't start with 3

$n(\text{start } 3) = 1 \times 10 \times 10 \times 10 = 1000$

$n(\text{don't start } 3) = n(S) - n(\text{start } 3)$
$= 9000$

$P(\text{doesn't start } 3) = \frac{9000}{10000}$
$= 0.9$

c) the number of PIN's that start with 0 or end with 99

$n(\text{start } 0 \text{ or end } 99) = n(\text{start } 0) + n(\text{end } 99)$
$- n(\text{start } 0 \text{ and end } 99)$

$= 1 \times 10 \times 10 \times 10 + 10 \times 10 \times 1 \times 1$

$- 1 \times 10 \times 1 \times 1$

$= 1000 + 100 - 10$

$= 1090$