1. [4 marks] Evaluate $\int \frac{5dx}{(1+x^2)\sqrt[3]{\tan^{-1} x}}$

$$= \int \frac{5(\tan^{-1}x)^{-1/3}\,dx}{1+x^2} \quad ①$$

$$\boxed{\begin{aligned} u &= \tan^{-1} x \\ du &= \frac{1}{1+x^2}\,dx \end{aligned}}$$

$$= \int 5u^{-1/3}\,du \quad ①$$

$$= 5\left(\tfrac{3}{2}u^{2/3}\right) + C \quad ①$$

$$= \tfrac{15}{2}(\tan^{-1}x)^{2/3} + C \quad ①$$

2. [3 marks] Evaluate $\int \frac{e^{3x}\,dx}{\csc(e^{3x})}$

$$= \int \sin(e^{3x}) \cdot e^{3x}\,dx$$

$$\boxed{\begin{aligned} u &= e^{3x} \\ du &= 3e^{3x}\,dx \\ e^{3x}\,dx &= \frac{du}{3} \end{aligned}}$$

$$= \tfrac{1}{3}\int \sin u\,du \quad ①$$

$$= -\tfrac{1}{3}\cos u + C \quad ①$$

$$= -\tfrac{1}{3}\cos(e^{3x}) + C \quad ①$$

3. [4 marks] Evaluate $\int_0^{\pi/36} (\sec^2 9x)\, e^{-\tan 9x}\, dx$

$u = -\tan 9x$

$du = -9\sec^2 9x\, dx$

$\sec^2 9x\, dx = \frac{-du}{9}$

When $x = 0$ $u = 0$

$x = \frac{\pi}{36}$ $u = -\tan\left(\frac{9\pi}{36}\right)$

$= -\tan\left(\frac{\pi}{4}\right)$

$= -1$

$= \frac{-1}{9}\int_0^{-1} e^u\, du$ 2

$= \frac{-1}{9}\left[e^u\right]_0^{-1}$ 1

$= \frac{-1}{9}\left[e^{-1} - 1\right]$ 1

or $\frac{1 - e^{-1}}{9}$

4. [3 marks] Evaluate $\int \frac{7x}{\sqrt{9-x^4}}\,dx$

$$= \int \frac{7x\,dx}{\sqrt{9-(x^2)^2}}$$

$$u = x^2$$
$$du = 2x\,dx$$
$$7x\,dx = \frac{7}{2}\,du$$

$$= \frac{7}{2}\int \frac{du}{\sqrt{9-u^2}} \quad 1$$

$$= \frac{7}{2}\sin^{-1}\frac{u}{3} + C \quad 1$$

$$= \frac{7}{2}\sin^{-1}\frac{x^2}{3} + C \quad 1$$

5. [3 marks] Evaluate $\int \frac{13dx}{(2x+1)\ln(2x+1)}$

$$u = \ln(2x+1)$$
$$du = \frac{2}{2x+1}\,dx$$
$$\frac{13dx}{2x+1} = \frac{13}{2}\,du$$

$$= \frac{13}{2}\int \frac{du}{u} \quad 1$$

$$= \frac{13}{2}\ln|u| + C \quad 1$$

$$= \frac{13}{2}\ln|\ln(2x+1)| + C \quad 1$$

6. [4 marks] Use Integration by Parts to evaluate $\int x^6 \ln x \; dx$

	D	I	
(u)	$\ln x$	x^6	(dv)
(du)	$\frac{1}{x}$	$x^7/7$	(v)

2

$$\int u\,dv = uv - \int v\,du$$

$$\int x^6 \ln x \, dx = \frac{x^7}{7} \ln x - \int \frac{x^6}{7} dx$$

$$= \frac{x^7}{7} \ln x - \frac{x^7}{49} + C$$

2

7. [4 marks] Use Partial Fractions to evaluate $\int \frac{4x^2+7}{x^3-x}\,dx$

$$x^3 - x = x(x^2-1)$$
$$= x(x-1)(x+1)$$

$$\frac{4x^2+7}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1}$$ 1

$$4x^2+7 = A(x-1)(x+1) + Bx(x+1) + Cx(x-1)$$

Sub $x=0$: $7 = -A$ $\boxed{A=-7}$

$x=1$: $11 = 2B$ $\boxed{B = 11/2}$ 1

$x=-1$: $11 = 2C$ $\boxed{C = 11/2}$

$$\int \frac{4x^2+7}{x^3-x}\,dx = \int \left(\frac{-7}{x} + \frac{11}{2}\cdot\frac{1}{x-1} + \frac{11}{2}\cdot\frac{1}{x+1} \right) dx$$
$$= -7\ln|x| + \frac{11}{2}\ln|x-1| + \frac{11}{2}\ln|x+1| + C$$

2

2/4 if you wrote $\frac{4x^2+7}{x^3-x} = \frac{A}{x} + \frac{B}{x^2-1}$