22 HYPERGEOMETRIC

Defective (D)	Good (G)
$800(0.04) = 32$	$800 - 32 = 768$

$n(S) = {}^{800}C_{20}$

$n(1 \text{ to } 3\ D) = n(1D) + n(2D) + n(3D)$

$= n(1D \text{ and } 19G)$

$+ n(2D \text{ and } 18G)$

$+ n(3D \text{ and } 17G)$

$= {}^{32}C_1 \times {}^{768}C_{19}$

$+ {}^{32}C_2 \times {}^{768}C_{18}$

$+ {}^{32}C_3 \times {}^{768}C_{17}$

$\approx 2.073 \times 10^{39}$

$$P(1 \text{ to } 3D) = \frac{2.073 \times 10^{39}}{{}^{800}C_{20}}$$

≈ 0.56

$\rightarrow$

Alternatively: $N = 800$

$n = 20$

Approximately binomial, since $\frac{N}{n} \geq 20$

X = # defective parts selected

$n = 20 \quad p = 0.04 \quad q = 1 - p = 0.96$

$$P(1 \leq X \leq 3) = P(X=1) + P(X=2) + P(X=3)$$

$$= {}_{20}C_1 (0.04)^1 (0.96)^{19}$$

$$+ {}_{20}C_2 (0.04)^2 (0.96)^{18}$$

$$+ {}_{20}C_3 (0.04)^3 (0.96)^{17}$$

$$\simeq 0.55$$

(23) X = # calls in next 3 mins

Poisson $\dfrac{1.2 \text{ calls}}{\text{min}} = \dfrac{3.6 \text{ calls}}{3 \text{ mins}}$

Use $\mu = 3.6$

$$P(X \geq 4) = 1 - P(X=0) - P(X=1) - P(X=2) - P(X=3)$$

$$= 1 - e^{-3.6}\left(\frac{3.6^0}{0!} + \frac{3.6^1}{1!} + \frac{3.6^2}{2!} + \frac{3.6^3}{3!}\right)$$

$$\approx 0.48$$

(24)

$$\mu \text{ or } E(X) = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \cancel{\int_{-\infty}^{1} 0\,dx} + \int_{1}^{2} \frac{x^3}{4}\,dx$$

$$+ \int_{2}^{4} \frac{5x^2}{72}\,dx + \cancel{\int_{4}^{\infty} 0\,dx}$$

$$= \left[\frac{x^4}{16}\right]_1^2 + \left[\frac{5x^3}{216}\right]_2^4$$

$$= \left(\frac{16}{16} - \frac{1}{16}\right) + \left(\frac{320}{216} - \frac{40}{216}\right)$$

$$= \frac{965}{432}$$

$$E(X^2) = \cancel{\int_{-\infty}^{1} 0\,dx} + \int_{1}^{2} \frac{x^4}{4}\,dx + \int_{2}^{4} \frac{5x^3}{72}\,dx + \cancel{\int_{4}^{\infty} 0\,dx}$$

$$= \left[\frac{x^5}{20}\right]_1^2 + \left[\frac{5x^4}{288}\right]_2^4$$

$$= \left(\frac{32}{20} - \frac{1}{20}\right) + \left(\frac{1280}{288} - \frac{80}{288}\right)$$

$$= \frac{4116}{720}$$

$\rightarrow$

Variance $\sigma^2 = E(X^2) - \mu^2$

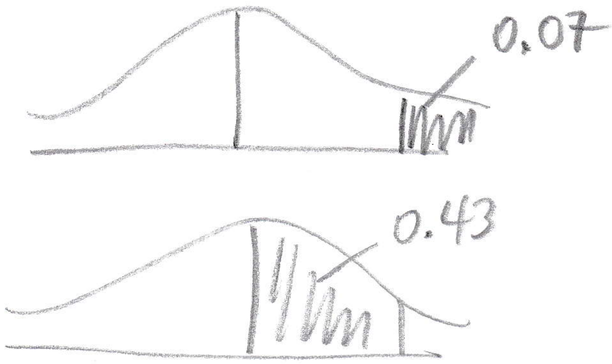
$$= \frac{4116}{720} - \left(\frac{965}{432}\right)^2$$

$$\approx 0.7268$$

SD $\sigma = \sqrt{0.7268}$

$$\approx 0.85$$

(25) X is normal $\mu=150$ $\sigma=35$.



Reverse look-up area $= 0.43$

$z = 1.48$

$$z = \frac{X-\mu}{\sigma}$$

$$1.48 = \frac{X-150}{35}$$

$$1.48(35) = X-150$$

$$X = 150 + 1.48(35)$$

$$X \approx 202$$

(26) $\mu = 67 \quad \sigma = 4 \quad n = 45$

$P(65.5 \le \bar{x} \le 66)$

$n \geq 30$ ✓

$z = \dfrac{\bar{x} - \mu}{(\sigma/\sqrt{n})}$

$z_1 = \dfrac{65.5 - 67}{(4/\sqrt{45})} \approx -2.52$

$z_2 = \dfrac{66 - 67}{(4/\sqrt{45})} \approx -1.68$

$= P(-2.52 \le z \le -1.68)$

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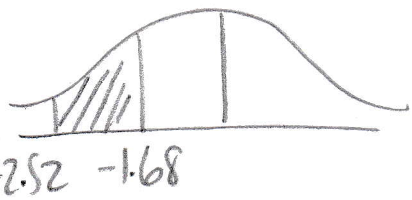

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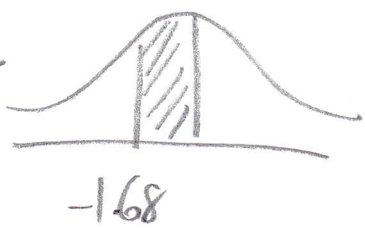

$= 0.4941 - 0.4535$

$= 0.0406$

27 $\sigma = 0.08$ $n = 80$ $\bar{x} = 11.03$

$\boxed{n \geq 30 \text{ USE } z}$

a) $\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

$\begin{cases} 1-\alpha = 0.98 \\ z_{\alpha/2} = 2.326 \end{cases}$

$11.03 \pm 2.326 \left(\frac{0.08}{\sqrt{80}} \right)$

$\boxed{11.01 \leq \mu \leq 11.05 \text{ inches}}$

b) $\bar{x} + z_{\alpha} \frac{\sigma}{\sqrt{n}}$

$\begin{cases} 1-\alpha = 0.9 \\ z_{\alpha} = 1.282 \end{cases}$

$= 11.03 + 1.282 \left(\frac{0.08}{\sqrt{80}} \right)$

≈ 11.04

$\boxed{\mu \leq 11.04 \text{ inches}}$

c) $\bar{x} - z_{\alpha} \frac{\sigma}{\sqrt{n}}$

$\begin{cases} 1-\alpha = 0.95 \\ z_{\alpha} = 1.645 \end{cases}$

$= 11.03 - 1.645 \left(\frac{0.08}{\sqrt{80}} \right)$

≈ 11.02

$\boxed{11.02 \text{ inches} \leq \mu}$

28

$\sigma = 5.6$

99% ME < 1.5

$z_{\alpha/2} \frac{\sigma}{\sqrt{n}} < 1.5$

$\begin{cases} 1-\alpha = 0.99 \\ z_{\alpha/2} = 2.576 \end{cases}$

$2.576 \left(\frac{5.6}{\sqrt{n}}\right) < 1.5$

$\frac{2.576(5.6)}{1.5} < \sqrt{n}$

Square both sides :

$\left[\frac{2.576(5.6)}{1.5}\right]^2 < n$

$92.4\ldots < n$

$\boxed{n = 93 \text{ is the minimum sample size}}$

29 $n=15$ $\bar{x}=37.2$ $s=4.6$

normal population

$\boxed{\begin{array}{l} n<30 \\ \text{normal pop} \\ \sigma \text{ unknown} \end{array}}$ use t

a) $\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$

$df = n-1 = 14$

$1-\alpha = 0.99$

$\alpha = 0.01$

$t_{\alpha/2} = t_{0.005} = 2.977$

$37.2 \pm 2.977\left(\frac{4.6}{\sqrt{15}}\right)$

$\boxed{33.7 \le \mu \le 40.7 \text{ ppm}}$

b) $\bar{x} + t_{\alpha} \frac{s}{\sqrt{n}}$

$df = n-1 = 14$

$1-\alpha = 0.9$

$\alpha = 0.1$

$t_{\alpha} = t_{0.1} = 1.345$

$= 37.2 + 1.345\left(\frac{4.6}{\sqrt{15}}\right)$

≈ 38.8

$\boxed{\mu \le 38.8 \text{ ppm}}$

c) $\bar{x} - t_\alpha \frac{s}{\sqrt{n}}$

$$\begin{cases} df = n-1 = 14 \\ 1-\alpha = 0.95 \\ \alpha = 0.05 \\ t_\alpha = 1.761 \end{cases}$$

$= 37.2 - 1.761 \left(\frac{4.6}{\sqrt{15}}\right)$

≈ 35.1

$\boxed{35.1 \text{ ppm} \leq \mu}$

(30) a) The association is positive
(as x increases, y increases)

$r^2 = 0.5477$

$r = \pm\sqrt{0.5477}$

Choose (+) since association is positive

$r \approx 0.74$

b) 54.77%

c) Sub $x=50$:

$y = 0.61(50) + 112.59$

$y = 143.09$

≈ 143

d) Sub $x = 70$:

$y = 0.61(70) + 112.59$

$= 155.29$

≈ 155

e) c) is valid while d) is invalid
We should not predict outside the interval of x-values, that is $46 \le x \le 65$

→

f) Sub $y = 145$

$$145 = 0.61x + 112.59$$

$$32.41 = 0.61x$$

$$x = \frac{32.41}{0.61}$$

$$\approx 53$$