(13) $(\cos x)\frac{dy}{dx} = 2 - y\sin x$

$(\cos x)\,dy = 2dx - y\sin x\,dx$

$dy = \frac{2}{\cos x}dx - \frac{y\sin x}{\cos x}dx$

$\boxed{dy + y\tan x\,dx = 2\sec x\,dx}$

Linear

$P(x) = \tan x$

$\int P(x)dx = \ln(\sec x)$

$I.F. = e^{\ln(\sec x)} = \sec x$

$\sec x\,dy + y\sec x\tan x\,dx = 2\sec^2 x\,dx$

Integrate:

$\boxed{\sec x \cdot y = 2\tan x + C}$ Implicit

Multiply by $\cos x$:

$\boxed{y = 2\sin x + C\cos x}$ Explicit

(14)

$m \cdot a = F_{net}$

$m\,a = F_g - F_{drag}$

$\left\{ \begin{array}{l} m = 1.00\ kg \\ F_g = 4.00\ N \\ F_{drag} = v \end{array} \right.$

$v(0) = 0$

$v(4) = ?$

$a = 4 - v$

$\boxed{\frac{dv}{dt} = 4 - v}$

$\frac{dv}{4-v} = dt$ $\boxed{\text{Separable}}$

$\int \frac{dv}{4-v} = \int dt$

$-\ln(4-v) = t + C_1$

$\ln(4-v) = -t + C_2$

$e^{LS} = e^{RS}$: $4 - v = e^{-t+C_2} \leftarrow e^{-t} \cdot \cancel{e^{C_2}}\ C_3$

$4 - v = C_3 e^{-t}$

$\boxed{4 + C_4 e^{-t} = v}$

Sub $t=0$, $v=0$: $4 + C_4 = 0$

$C_4 = -4$

$\boxed{v = 4 - 4e^{-t}}$

Sub $t=4$: $v = 4 - 4e^{-4}$

$\approx 3.93\ m/s$

15 a) $y'' + 6y' + 8y = 0$

$m^2 + 6m + 8 = 0$

$(m+2)(m+4) = 0$

$m = -2, -4$ Distinct Real Roots

$y = C_1 e^{-2x} + C_2 e^{-4x}$

b) $y'' + 6y' + 9y = 0$

$m^2 + 6m + 9 = 0$

$(m+3)^2 = 0$

$m = -3, -3$ Repeated Real Roots

$y = C_1 e^{-3x} + C_2 x e^{-3x}$

or $y = (C_1 + C_2 x) e^{-3x}$

c) $y'' + 6y' + 10y = 0$

$m^2 + 6m + 10 = 0$

$$m = \frac{-6 \pm \sqrt{36 - 40}}{2}$$

$$m = \frac{-6 \pm \sqrt{-4}}{2} \Leftarrow \sqrt{4}\sqrt{-1}$$

$$m = \frac{-6 \pm 2j}{2}$$

$m = -3 \pm j$ Complex Roots

$\alpha = -3 \quad \beta = 1$

$$y = e^{-3x}(C_1 \sin x + C_2 \cos x)$$

16

a) $y_p = \cancel{Ae^{5x}} + Be^{6x}$ BAD CASE

$y_p = Axe^{5x} + Be^{6x}$

b) $y_p = Ax\sin x + Bx\cos x$
$+ C\sin x + D\cos x$

not bad case

c) $y_p = \cancel{A} + Be^{-2x}$ BAD CASE

$y_p = Ax + Be^{-2x}$

(17)

$$\frac{d^2y}{dx^2} + 4y = 2\sin x \qquad y(\pi) = 14 \quad y'(\pi) = -8$$

1) Find y_c

$m^2 + 4 = 0$

$m = \frac{\pm\sqrt{-16}}{2} \leftarrow \sqrt{16}\sqrt{-1}$

$m = \frac{\pm 4j}{2}$

$m = \pm 2j$

$\boxed{\begin{array}{c}\text{Complex Roots} \\ \alpha = 0 \quad \beta = 2\end{array}}$

$y_c = e^{0x}(C_1\sin 2x + C_2\cos 2x)$

$y_c = C_1\sin 2x + C_2\cos 2x$

2) Find y_p

$y_p = A\sin x + B\cos x$ <u>not bad case</u>

3) $y_p \to DE$

$$\begin{cases} y_p = A\sin x + B\cos x \\ y_p' = A\cos x - B\sin x \\ y_p'' = -A\sin x - B\cos x \end{cases}$$

$$y'' + 4y = 2\sin x$$

$$-A\sin x - B\cos x + 4(A\sin x + B\cos x) = 2\sin x$$

$$3A\sin x + 3B\cos x = 2\sin x + 0\cos x$$

$$3A = 2 \qquad A = 2/3$$

$$3B = 0 \qquad B = 0$$

$$y_p = \frac{2}{3}\sin x$$

4) $y = y_c + y_p$

$$\boxed{y = C_1\sin 2x + C_2\cos 2x + \frac{2}{3}\sin x}$$

5) Initial Conditions

$x = \pi$, $y = 14$: $\quad 14 = 0 + C_2(1) + 0$

$$\boxed{C_2 = 14}$$

$$\boxed{y = C_1\sin 2x + 14\cos 2x + \frac{2}{3}\sin x}$$

$$y' = 2C_1\cos 2x - 28\sin 2x + \frac{2}{3}\cos x$$

$x = \pi$, $y' = -8$: $\quad -8 = 2C_1(1) + 0 + \frac{2}{3}(-1)$

$$-8 + \frac{2}{3} = 2C_1$$

$$\frac{-22}{3} = 2C_1$$

$C_1 = \frac{-11}{3}$

$y = \frac{-11}{3} \sin 2x + 14 \cos 2x + \frac{2}{3} \sin x$

18

$$m\frac{d^2x}{dt^2} = -\beta\frac{dx}{dt} - kx + f(t)$$

$\ominus$ $x(0) = -0.5$

$\oplus$ $x'(0) = 0.1$

$$3\frac{d^2x}{dt^2} = -18\frac{dx}{dt} - 24x + t$$

$$\boxed{3\frac{d^2x}{dt^2} + 18\frac{dx}{dt} + 24x = t}$$

1) Find x_c

$$3n^2 + 18n + 24 = 0$$

$$n^2 + 6n + 8 = 0$$

$$(n+4)(n+2) = 0$$

$$n = -4, -2$$

$$x_c = C_1e^{-4t} + C_2e^{-2t}$$ ← Note: x is a function of t

2) $x_p = At + B$ <u>not bad case</u>

3) $x_p \to$ DE

$$\begin{cases} x_p = At + B \\ x_p' = A \\ x_p'' = 0 \end{cases}$$

$$3x'' + 18x' + 24x = t$$

$$3(0) + 18(A) + 24(At+B) = t$$

$$24At + (18A + 24B) = t + 0$$

$$24A = 1 \qquad A = \frac{1}{24}$$

$$18A + 24B = 0$$

$$18\left(\frac{1}{24}\right) + 24B = 0$$

$$24B = \frac{-18}{24}$$

$$B = \frac{-18}{576} = \frac{-1}{32}$$

$$\boxed{x_p = \frac{t}{24} - \frac{1}{32}}$$

4) $x = x_c + x_p$

$$\boxed{x = C_1 e^{-4t} + C_2 e^{-2t} + \frac{t}{24} - \frac{1}{32}}$$

5) Initial Conditions

$t = 0$
$x = -0.5$:

$$-0.5 = C_1 + C_2 - \frac{1}{32}$$

$$C_1 + C_2 = -0.5 + \frac{1}{32}$$

$$C_1 + C_2 = \frac{-15}{32} \quad (1)$$

$$x' = -4C_1 e^{-4t} - 2C_2 e^{-2t} + \frac{1}{24}$$

$t=0$
$x'=0.1$:

$$0.1 = -4C_1 - 2C_2 + \frac{1}{24}$$

$$-4C_1 - 2C_2 = 0.1 - \frac{1}{24}$$

$$-4C_1 - 2C_2 = \frac{7}{120} \quad (2)$$

$2 \times (1)$: $\quad 2C_1 + 2C_2 = \frac{-15}{16}$

(2): $\quad -4C_1 - 2C_2 = \frac{7}{120}$

$+$

$$-2C_1 = \frac{-211}{240}$$

$$C_1 = \frac{211}{480}$$

$C_1 = \frac{211}{480} \rightarrow (1)$: $\quad \frac{211}{480} + C_2 = \frac{-15}{32}$

$$C_2 = \frac{-15}{32} - \frac{211}{480}$$

$$= \frac{-436}{480}$$

$$= \frac{-109}{120}$$

$$\boxed{x = \frac{211}{480} e^{-4t} - \frac{109}{120} e^{-2t} + \frac{t}{24} - \frac{1}{32}}$$

(19) # of measurements $n=4$

a) pop. mean $\mu = \frac{68+72+55+41}{4} = 59$

b) ordered: 41, 55, 68, 72

median $= \frac{55+68}{2} = 61.5$

c)

X	$X-\mu$	$(X-\mu)^2$
68	9	81
72	13	169
55	-4	16
41	-18	324

$\mu = 59$

Pop. variance

$\sigma^2 = \frac{\text{Sum of } (X-\mu)^2}{n}$

$= \frac{590}{4}$

$= 147.5$

Pop SD $\sigma = \sqrt{147.5}$

≈ 12

d) The spread will decrease

So $\sigma^2_{new} < \sigma^2_{old}$. Conclude $\sigma^2_{new} < 147.5$

(20)

	B good	B defective
A good	0.975	
A defective		

$$P(A\ good) = 1 - P(A\ defective) = 1 - 0.023 = 0.977$$

	B good	B def
A good	0.975	0.002 ← 0.977 − 0.975
A def		

	B good	B def
A good	0.975	0.002
A def		0.017 ↑ 0.019 − 0.002

For completeness:

	B good	B def
A good	0.975	0.002
A def	0.006	0.017

0.006 ← 1 − 0.975 − 0.002 − 0.017

$$\boxed{P(both\ defective) = 0.017}$$

21 a) $P(X \leq 4) = P(X=-2) + P(X=3)$
$= 0.1 + 0.6$
$= 0.7$

b) μ or $E(X) = \sum x \, P(x)$
$= -2(0.1) + 3(0.6) + 7(0.3)$
$= 3.7$

c) $E(X^2) = \sum x^2 P(x)$
$= (-2)^2(0.1) + 3^2(0.6) + 7^2(0.3)$
$= 20.5$

$\sigma^2 = E(X^2) - \mu^2$
$= 20.5 - 3.7^2$
$= 6.81$

d) $\sigma = \sqrt{6.81}$
≈ 2.6

e) $P(\mu - 1.5\sigma \leq X \leq \mu + 1.5\sigma)$
$= P(-0.2 \leq X \leq 7.6)$
$= P(X=3) + P(X=7)$
$= 0.6 + 0.3$
$= 0.9$