① $\int_0^2 x e^{7x^2} dx$

$u = 7x^2$

$du = 14x\,dx$

$x\,dx = du/14$

When $x=0$ $u=0$

$x=2$ $u=28$

$= \frac{1}{14}\int_0^{28} e^u du$

$= \frac{1}{14}\left[e^u\right]_0^{28}$

$= \frac{1}{14}(e^{28} - e^0)$

$= \frac{1}{14}(e^{28} - 1)$

② $\int (x^2-3)e^{7x}\,dx$

Integration by Parts

	D	I
$+$	x^2-3	e^{7x}
$-$	$2x$	$e^{7x}/7$
$+$	2	$e^{7x}/49$
		$e^{7x}/343$

$$\int (x^2-3)e^{7x}\,dx = \frac{(x^2-3)e^{7x}}{7} - \frac{2xe^{7x}}{49} + \frac{2e^{7x}}{343} + C$$

$$= \left(\frac{x^2}{7} - \frac{2x}{49} - \frac{145}{343}\right)e^{7x} + C$$

③ $\displaystyle\int \frac{t}{\sqrt[3]{6t^2+1}}\,dt$

$$\boxed{\begin{aligned} u &= 6t^2+1 \\ du &= 12t\,dt \\ t\,dt &= du/12 \end{aligned}}$$

$$= \frac{1}{12}\int \frac{du}{\sqrt[3]{u}}$$

$$= \frac{1}{12}\int u^{-1/3}\,du$$

$$= \frac{1}{12}\left(\frac{3}{2}u^{2/3}\right)+c$$

$$= \frac{1}{8}(6t^2+1)^{2/3}+c$$

4 $\displaystyle\int \frac{6e^{3x}}{1+4e^{3x}}\,dx$

$u = 1+4e^{3x}$

$du = 12e^{3x}\,dx$

$6e^{3x}\,dx = du/2$

$= \frac{1}{2}\int \frac{du}{u}$

$= \frac{1}{2}\ln|u| + C$

$= \frac{1}{2}\ln\left|1+4e^{3x}\right| + C$

(5) $\int \ln x \, dx$

Integration by Parts

	D	I	
(u)	$\ln x$	1	(dv)
(du)	$\frac{1}{x}$	x	(v)

$$\int u\,dv = uv - \int v\,du$$

$$\int \ln x\,dx = x\ln x - \int 1\,dx$$

$$= x\ln x - x + C$$

⑥ $\int \frac{7x}{\sqrt{5-9x^4}}\,dx$

$= \int \frac{7x}{\sqrt{5-(3x^2)^2}}\,dx$

$$\boxed{\begin{aligned} u &= 3x^2 \\ du &= 6x\,dx \\ x\,dx &= \tfrac{1}{6}\,du \end{aligned}}$$

$= \frac{7}{6}\int \frac{du}{\sqrt{5-u^2}}$

$= \frac{7}{6}\int \frac{du}{\sqrt{\sqrt{5}^2-u^2}}$

$= \frac{7}{6}\sin^{-1}\left(\frac{u}{\sqrt{5}}\right)+C$

$= \frac{7}{6}\sin^{-1}\left(\frac{3x^2}{\sqrt{5}}\right)+C$

⑦ $\int \frac{\csc(\tan^{-1}x)\cot(\tan^{-1}x)}{1+x^2}\,dx$

$$\boxed{\begin{aligned} u &= \tan^{-1}x \\ du &= \frac{dx}{1+x^2} \end{aligned}}$$

$= \int \csc u \cot u \, du$

$= -\csc u + C$

$= -\csc(\tan^{-1}x) + C$

(8) $\int_1^3 \frac{x^2-x}{x^2-25}\,dx$

Long Division

$$\begin{array}{r} 1 \\ (x^2-25)\overline{)\,x^2-x+0} \\ -(x^2 \quad -25) \\ \hline -x+25 \end{array}$$

$$= \int_1^3 \left(1 + \frac{-x+25}{x^2-25}\right)dx$$

$$= \int_1^3 \left(1 + \frac{-x+25}{(x-5)(x+5)}\right)dx$$

Partial Fractions

$$\frac{-x+25}{(x-5)(x+5)} = \frac{A}{x-5} + \frac{B}{x+5}$$

$$-x+25 = A(x+5) + B(x-5)$$

Sub $x=5$: $20 = 10A$ $\quad A=2$

Sub $x=-5$: $30 = -10B$ $\quad B=-3$

$$= \int_1^3 \left(1 + \frac{2}{x-5} - \frac{3}{x+5}\right)dx$$

$\rightarrow$

$$= \left[x + 2\ln|x-5| - 3\ln|x+5|\right]_1^3$$

$$= (3 + 2\ln|-2| - 3\ln 8)$$

$$- (1 + 2\ln|-4| - 3\ln 6)$$

$$= 3 + 2\ln 2 - 3\ln 8 - 1 - 2\ln 4 + 3\ln 6$$

$$= 2 + \ln 4 - \ln 512 - \ln 16 + \ln 216$$

$\boxed{\text{using } n\ln a = \ln a^n}$

$$= 2 + \ln 864 - \ln 8192$$

$\boxed{\text{using } \ln a + \ln b = \ln(ab)}$

$$= 2 + \ln \frac{864}{8192}$$

$\boxed{\text{using } \ln a - \ln b = \ln\left(\frac{a}{b}\right)}$

or $2 + \ln \frac{27}{256}$

(9) $f = y\cos x + xe^y + 5xy$

$$\frac{\partial f}{\partial x} = -y\sin x + e^y + 5y$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right)$$

$$= \frac{\partial}{\partial x}\left(-y\sin x + e^y + 5y\right)$$

$$= -y\cos x$$

$$\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)$$

$$= \frac{\partial}{\partial y}\left(-y\sin x + e^y + 5y\right)$$

$$= -\sin x + e^y + 5$$

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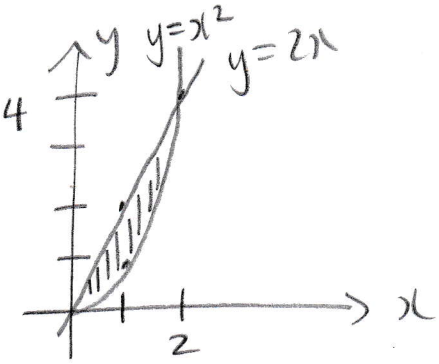

$$\boxed{\begin{aligned} x^2 \le\ & y \le 2x \\ 0 \le\ & x \le 2 \end{aligned}}$$

$$V = \iint z \, dy \, dx$$

$$= \int_0^2 \int_{x^2}^{2x} z \, dy \, dx$$

$$= \int_0^2 \int_{x^2}^{2x} (x+y) \, dy \, dx$$

$$= \int_0^2 \left[xy + \frac{y^2}{2} \right]_{y=x^2}^{y=2x} dx$$

$$= \int_0^2 \left[\left(2x^2 + \frac{4x^2}{2}\right) - \left(x^3 + \frac{(x^2)^2}{2}\right) \right] dx$$

$$= \int_0^2 \left(4x^2 - x^3 - \frac{x^4}{2}\right) dx$$

$$= \left[\frac{4x^3}{3} - \frac{x^4}{4} - \frac{x^5}{10}\right]_0^2$$

$$= \left(\frac{4(8)}{3} - \frac{16}{4} - \frac{32}{10}\right) - 0$$

$$= \frac{104}{30} \quad \text{or} \quad \frac{52}{15}$$

(11)

$$\boxed{y = x\ln x - Cx}$$

$$y' = x\left(\frac{1}{x}\right) + \ln x\,(1) - C$$

$$\boxed{y' = 1 + \ln x - C}$$

$$LS = x + y - xy'$$
$$= x + (x\ln x - Cx) - x(1 + \ln x - C)$$
$$= x + x\ln x - Cx - x - x\ln x + Cx$$
$$= 0$$
$$= RS$$

(12) $(x^3-2)^4 dy + 27x^2 dx = 0$

$$dy + \frac{27x^2\,dx}{(x^3-2)^4} = 0$$ $\boxed{\text{Separable}}$

$$\int dy + \int \frac{27x^2\,dx}{(x^3-2)^4} = \int 0$$

$\boxed{\begin{aligned} \text{Sub } u &= x^3-2 \\ du &= 3x^2\,dx \\ 27x^2\,dx &= 9\,du \end{aligned}}$

$$\text{Integral} = \int \frac{9\,du}{u^4}$$

$$= -3u^{-3} + C_1$$

$$= -3(x^3-2)^{-3} + C_1$$

$$y - 3(x^3-2)^{-3} = C$$ Implicit Solution

$$\boxed{y = 3(x^3-2)^{-3} + C}$$ Explicit

Sub $y=5$, $x=2$: $5 = 3(6)^{-3} + C$

$$C = 5 - \frac{3}{6^3} = \frac{1077}{216} = \frac{359}{72}$$

$$\boxed{y = 3(x^3-2)^{-3} + \frac{359}{72}}$$