23 a) number of measurements $n=6$

Sample mean $\bar{x} = \frac{7+4+\ldots+1}{6} = 1.5$

b) in order : $-4, -2, 1, 3, 4, 7$

$\text{median} = \frac{1+3}{2} = 2$

c)

X	$X-\bar{x}$	$(X-\bar{x})^2$
7	5.5	30.25
4	2.5	6.25
-4	-5.5	30.25
-2	-3.5	12.25
3	1.5	2.25
1	-0.5	0.25

$\bar{x} = 1.5$

Sample Variance $s^2 = \frac{\text{sum of } (X-\bar{x})^2}{n-1}$

$= \frac{81.5}{5}$

$= 16.3$

Sample SD $\quad s = \sqrt{16.3}$

≈ 4

d) Adding a constant to each measurement does not affect the spread.

$s^2_{new} = s^2_{old} = 16.3$

(24) $n(S) = 45$

	Job	No Job
Live Alone	3	
Don't Live Alone		

	J	NJ
LA	3	5
DLA	28	

	J	NJ
LA	3	5
DLA	28	9

$\uparrow$ $45-3-5-28$

P(doesn't live alone and no job)

$$= \frac{n(\text{don't live alone and no job})}{n(S)}$$

$$= \frac{9}{45}$$

$$= 0.2$$

(25) # different symbols $= 52$

$n(S) = \underline{52} \times \underline{52} \times \ldots = 52^8$

a) $n(\text{end } yz) = \underline{52} \times \underline{52} \times \underline{\ } \ldots \underline{\ } \times \underline{52} \times \underline{1} \times \underline{1}$
$= 52^6$

$P(\text{end } yz) = \dfrac{52^6}{52^8}$
≈ 0.0004

b) $n(\text{start } b) = \underline{1} \times \underline{52} \times \ldots \times \underline{52}$
$= 52^7$

$n(\text{don't start } b) = n(S) - n(\text{start } b)$
$= 52^8 - 52^7$

$P(\text{don't start } b) = \dfrac{n(\text{don't start } b)}{n(S)}$

$= \dfrac{52^8 - 52^7}{52^8}$

≈ 0.98

→

c) n(start C or end C) = n(start C)

+ n(end C) − n(start C and end C)

$= 52^7 + 52^7 - \underline{1} \times \underline{52} \times \ldots \times \underline{52} \times \underline{1}$

$= 52^7 + 52^7 - 52^6$

$P(\text{start C or end C}) = \dfrac{n(\text{start C or end C})}{n(S)}$

$= \dfrac{52^7 + 52^7 - 52^6}{52^8}$

≈ 0.04

(26) a) X = earnings of Project A ($)

X	$P(X)$
8,000	0.65
3,000	0.15
-5,000	0.2

$$\mu \text{ or } E(X) = \sum x\, P(x)$$

$$= 8{,}000(0.65) + 3{,}000(0.15) - 5{,}000(0.2)$$

$$= \$4650$$

b) $$E(X^2) = \sum x^2 P(x)$$

$$= (8{,}000)^2(0.65) + (3{,}000)^2(0.15) + (-5{,}000)^2(0.2)$$

$$= 47{,}950{,}000$$

$$\sigma^2 = E(X^2) - \mu^2$$

$$= 47{,}950{,}000 - (4650)^2$$

$$= 26{,}327{,}500$$

$$\sigma \approx \$5131$$

c) Project B's earnings are more uncertain (more risky) since $\sigma_B > \sigma_A$.

27

$\dfrac{2}{D} \Big/ \dfrac{28}{G}$

D = defective
G = good

$\boxed{\text{HYPERGEOMETRIC}}$

$n(S) = {}_{30}C_5$

$n(\text{at least } 1D) = n(1D) + n(2D)$

$= n(1D \text{ and } 4G)$

$+ n(2D \text{ and } 3G)$

$= {}_2C_1 \times {}_{28}C_4 + {}_2C_2 \times {}_{28}C_3$

$= 44\,226$

$P(\text{at least } 1D) = \dfrac{n(\text{at least } 1D)}{n(S)}$

$= \dfrac{44226}{{}_{30}C_5}$

≈ 0.31

(28) X = # calls in next 2 mins

Poisson $\quad \dfrac{1.8 \text{ calls}}{\text{min}} = \dfrac{3.6 \text{ calls}}{2 \text{ mins}}$

$\mu = 3.6$

$$P(X \geq 3) = 1 - P(X=0) - P(X=1) - P(X=2)$$

$$= 1 - e^{-3.6}\left(\frac{3.6^0}{0!} + \frac{3.6^1}{1!} + \frac{3.6^2}{2!}\right)$$

$$= 0.70$$

(29) X = # questions student gets right

Binomial $n = 3$ $p = \frac{1}{4} = 0.25$ $q = 1 - p = 0.75$

X	$P(x) = {}_nC_x\, p^x q^{n-x}$
0	${}_3C_0 (0.25)^0 (0.75)^3 \approx 0.42$
1	${}_3C_1 (0.25)^1 (0.75)^2 \approx 0.42$
2	${}_3C_2 (0.25)^2 (0.75) \approx 0.14$
3	${}_3C_3 (0.25)^3 (0.75)^0 \approx 0.02$

30 a) $P(X=2.5)=0$

X is a continuous variable

b) $P(2.5 \le X \le 4.5) = \int_{2.5}^{4.5} f(x)\,dx$

$$= \int_{2.5}^{3} \frac{x^2}{20}\,dx + \int_{3}^{4.5} \frac{33}{2440}(x+x^2)\,dx$$

$$= \frac{x^3}{60}\Big|_{2.5}^{3} + \frac{33}{2440}\left(\frac{x^2}{2}+\frac{x^3}{3}\right)\Big|_{3}^{4.5}$$

$$= \frac{3^3}{60} - \frac{2.5^3}{60} + \frac{33}{2440}\left[\frac{4.5^2}{2} + \frac{4.5^3}{3} - \left(\frac{3^2}{2}+\frac{3^3}{3}\right)\right]$$

$$\approx 0.55$$

c) $P(X>4.5) = \int_{4.5}^{\infty} f(x)\,dx$

$$= \int_{4.5}^{5} \frac{33}{2440}(x+x^2)\,dx + \cancel{\int_{5}^{\infty} 0\,dx}$$

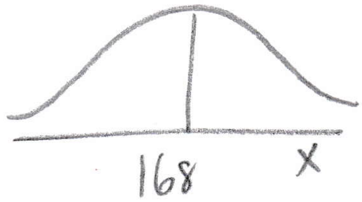
$$= \frac{33}{2440}\left(\frac{x^2}{2}+\frac{x^3}{3}\right)\Big|_{4.5}^{5}$$

$$= \frac{33}{2440}\left(\left(\frac{25}{2}+\frac{125}{3}\right) - \left(\frac{4.5^2}{2}+\frac{4.5^3}{3}\right)\right)$$

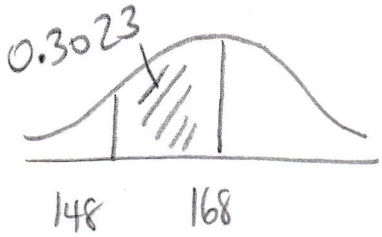
$$\approx 0.18$$

(31) X = weight (lbs)

X is normal $\quad \mu = 168$



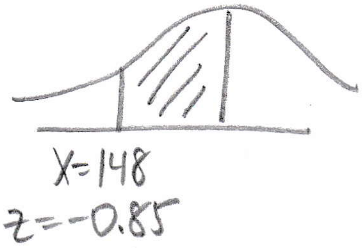
$\boxed{X \text{ is centred at } \mu}$



Reverse look-up area = 0.3023

Use negative z by symmetry

$\boxed{z = -0.85}$



Now $z = \dfrac{X - \mu}{\sigma}$

$$-0.85 = \frac{148 - 168}{\sigma}$$

$$\sigma = \frac{-20}{-0.85}$$

$\sigma \approx 24$ pounds

(32) $\mu = 71 \quad \sigma = 6 \quad n = 35$

$P(\bar{x} < 68 \text{ or } \bar{x} > 73)$

$n \geq 30$ ✓

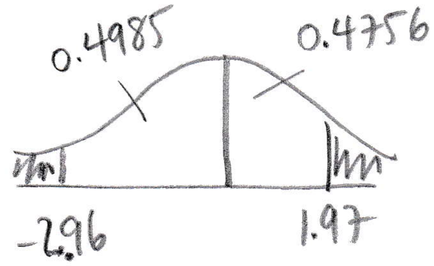
$z = \dfrac{\bar{x} - \mu}{(\sigma / \sqrt{n})}$

$z_1 = \dfrac{68 - 71}{(6/\sqrt{35})} \approx -2.96$

$z_2 = \dfrac{73 - 71}{(6/\sqrt{35})} \approx 1.97$

$= P(z < -2.96 \text{ or } z > 1.97)$

$=$



$= 1 - 0.4985 - 0.4756$

$= 0.0259$